FOSSEE, IIT Bombay
OpenFOAM Research Migration Project
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Numerical Modeling of Rayleigh Taylor Instability in Confined Geometries

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Synopsis

The Rayleigh-Taylor instability is an instability of the interface separating two immiscible fluids where denser fluid rests on the lighter one. In case of a disturbance of the interface, the dense fluid enters the light fluid. Although it has been an area of active research in fluid dynamics for the last twenty years, relatively little attention has been paid to the dynamics of problems where Rayleigh-Taylor instability plays a role but is only one component of a more complex system. Various real-life phenomena and applications of RT instability include explosions in supernovae, instabilities in liquid metal batteries, plasma fusion reactors, and inertial confinement fusion.

In this project, Rayleigh-Taylor instability between miscible fluids is to be examined in situations where it is confined by a cuboidal geometry. When the fluids are confined by a small geometry, it is known that there is a critical depth in which the RTI can be suppressed. The initial condition of the simulation will be that the interface is flat. By varying the geometrical parameters, the critical depth for a given geometry will be determined.

This project was migrated from the paper
Title: Confinement-induced stabilization of the Rayleigh-Taylor instability and transition to the unconfined limit
Authors: Samar Alqatari, Thomas E. Videbæk, Sidney R. Nagel, A. E. Hosoi, Irmgard Bischofberger
Journal name: Science Advances, Applied Science and Engineering

1. Introduction

The Rayleigh-Taylor instability arises when a dense fluid sinks and displaces a lighter one located below it. The paper looks at Rayleigh-Taylor instability with clean initial conditions under confinement. This adds an unexpected feature: Below a critical plate spacing, b_c, the Rayleigh- Taylor instability remarkably no longer occurs. This demonstrates the existence of an additional important length scale when nearby boundaries are present. Alqatari *et al.* analyze the competition between the destabilizing effect of buoyancy and the stabilizing effects of both momentum and mass diffusion. Their work also deals with instability in unconfined limits but that is not a part of this research migration project.

2. Governing Equations and Models

2.1 Continuity Equation

$$\frac{\partial u_j}{\partial x_j} = 0$$

2.2 Momentum Equation

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial(\tau_{tij} + \tau_{ij})}{\partial x_j} + \rho g_i + f_{\sigma i}$$

u - velocity
g - gravitational acceleration
p - pressure
τ_{ij} - viscous stresses
τ_{tij} - turbulent stresses
$f_{\sigma i}$ - surface tension

The density ρ is defined as follows:

$$\rho = \alpha\rho_1 + (1 - \alpha)\rho_2$$

Where ρ1 and ρ2 are densities of the two fluids. At the interphase between the two fluids α varies between 0 and 1.

The surface tension $f_{\sigma i}$, is modelled as continuum surface force (CSF). It is calculated as follows:

$$f_{\sigma i} = \sigma\kappa\frac{\partial\alpha}{\partial x_i}$$

σ - surface tension constant
κ - curvature.

The curvature can be approximated as follows:

$$\kappa = -\frac{\partial n_i}{\partial x_i} = -\frac{\partial}{\partial x_i}\left(\frac{\partial \alpha / \partial x_i}{|\partial \alpha / \partial x_i|}\right)$$

2.3 Interphase Equation

In order to know where the interphase between the two fluids is, an additional equation for α has to be solved.

$$\frac{\partial \alpha}{\partial t} + \frac{\partial (\alpha u_j)}{\partial x_j} = 0$$

3. Simulation Procedure

3.1 Geometry and Mesh

The geometry used was a Hele-Shaw box consisting of 2 parallel plates separated by a thin gap of size b. The cross-section of the Hele-Shaw cell is a rectangular domain of height b and width L = [10,16]b, large enough for the dynamics to be independent of the domain size. The top wall is type ‘patch’ named ‘topWall’. Rest 3 boundaries are type ‘wall’. The front and back patches are ‘empty’.

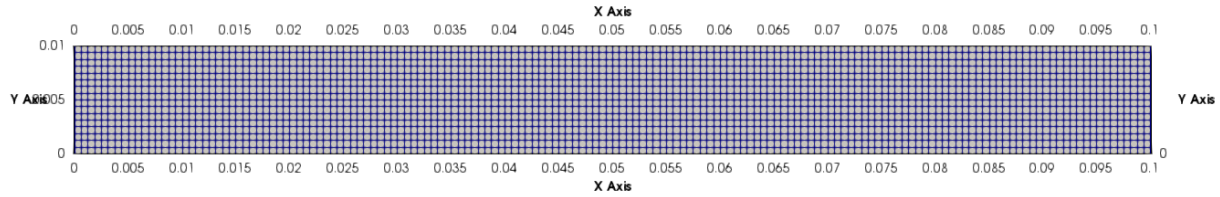


Fig. 1 z-normal view

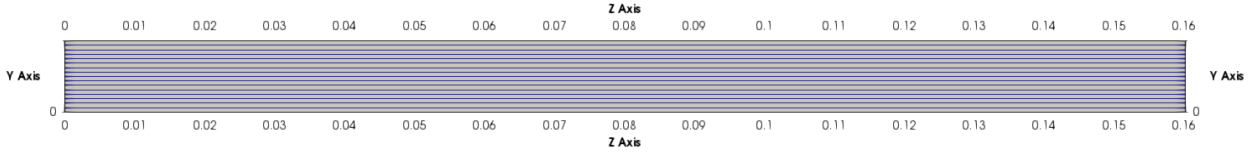


Fig. 2 x-normal view

As this is a 2D problem, mesh with simple grading (160 16 1) was used. Some simulations were also done with coarse mesh (80 8 1) to get preliminary results and conserve time.

3.2 Initial and Boundary Conditions

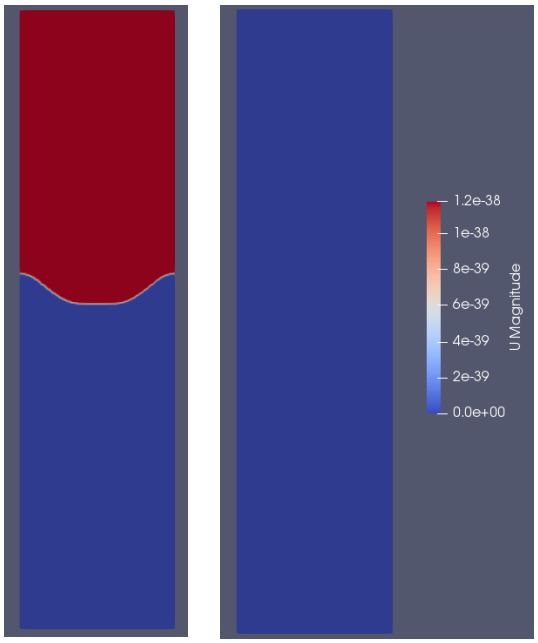
All the simulations were done in laminar regime. Initial condition is shown in Fig. 4 (a). The front and back patches are empty for all quantities.

Quantity	Boundary patch	Type
alpha (α)	topWall	zeroGradient
	walls	zeroGradient
p_rgh	topWall	fixedValue - 0
	walls	zeroGradient
velocity (U)	topWall	slip
	walls	slip

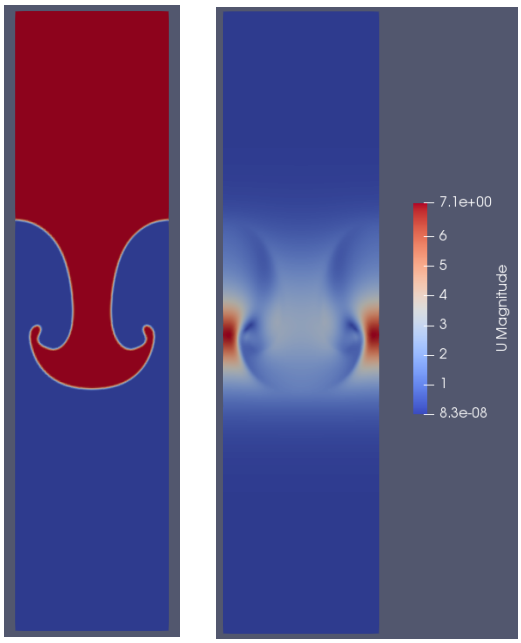
3.3 Solver

InterFoam with a laminar model has been used in this simulation. The PISO (Pressure Implicit with Splitting of Operators) loop was chosen as the solver algorithm since it works well for transient simulations and for low courant numbers. All the simulations were done using parallel processing with the 'simple' method. InterFoam uses a volume of fluid approach to solve for two incompressible immiscible fluids under isothermal conditions. The Navier Stokes equations for two incompressible, isothermal immiscible fluids are solved by the solver. Except for the interphase, the material properties are constant in the region filled by one of the two fluids. For the purpose of advecting the sharp interface, interFoam employs the OpenFOAM-specific algebraic VOF approach known as MULES (Multidimensional Universal Limiter with Explicit Solution).

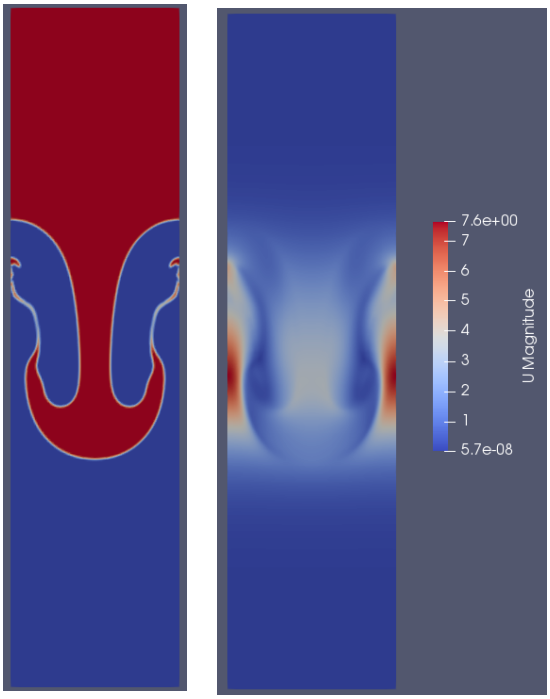
An initial simulation of RT instability was performed in a 1x4x1 box geometry and validated using [5]. The initial interface conditions were coded into the alpha.air initial file using codestream. The boundary conditions are the same as mentioned in the section above. The characteristic mushroom shape can be observed after time = 0.6 sec. The two fluids chosen were air-helium.



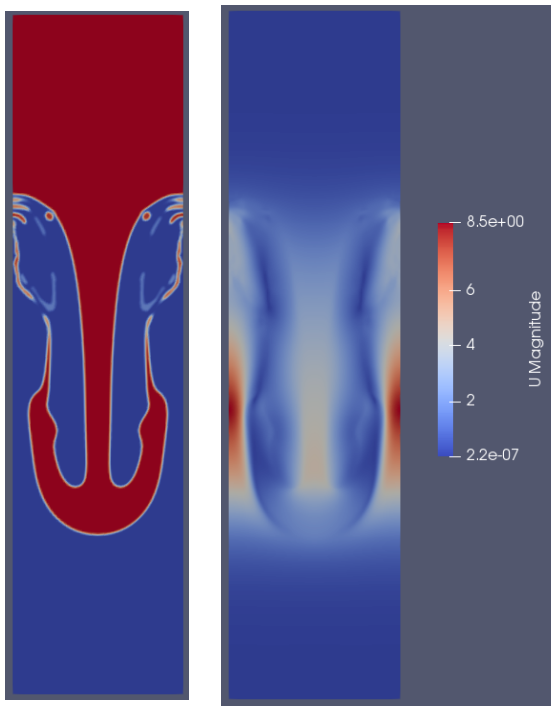
(a) t = 0s



(b) t = 0.6s



(c) t = 0.75s



(d) t = 1s

Fig.3 Air-helium volume fraction and velocity profile (air on top - red) at different time steps

4. Results and Discussions

The original plan was to find the critical height b_c at which RT instability does not occur for a range of fluid pairs with different orders of density differences. But as the density difference $\Delta\rho$ increases, b_c decreases; as buoyancy forces drive the instability. The simulations were done on an 8GB RAM system with 4 cores (8 processors). As the dimensions of geometry are decreased, timestep also decreases to keep the Courant number lower than 1. For $\Delta\rho \sim 10^{-3}$ g/cm^3 and b = 0.5mm, the timestep $\Delta t \sim 10^{-7}$ seconds. Even for a coarse mesh, due to limited system specifications, these simulations were taking a lot of time. Therefore the results presented here are for lower orders of density differences.

Heavier fluid	Density (kg/m^3)	Lighter fluid	Density (kg/m^3)	Average viscosity (10^{-5}m^2/s)	$\Delta\rho$ (g/cm^3)	Critical height b_c (mm)
Hydrogen	0.09	Methane	0.072	11.818	10^{-5}	6
Helium	0.1694	Hydrogen	0.09	10.815	10^{-5}	6
Oxygen	1.43	Nitrogen	1.25	1.3458	10^{-4}	~ 1
Air	1.255	Helium	0.1694	12.417	10^{-3}	Less than 0.5
Blood	1050	Water	1000	0.2643	10^{-2}	x
Water	1000	Oil	800	12.55	10^{-1}	x

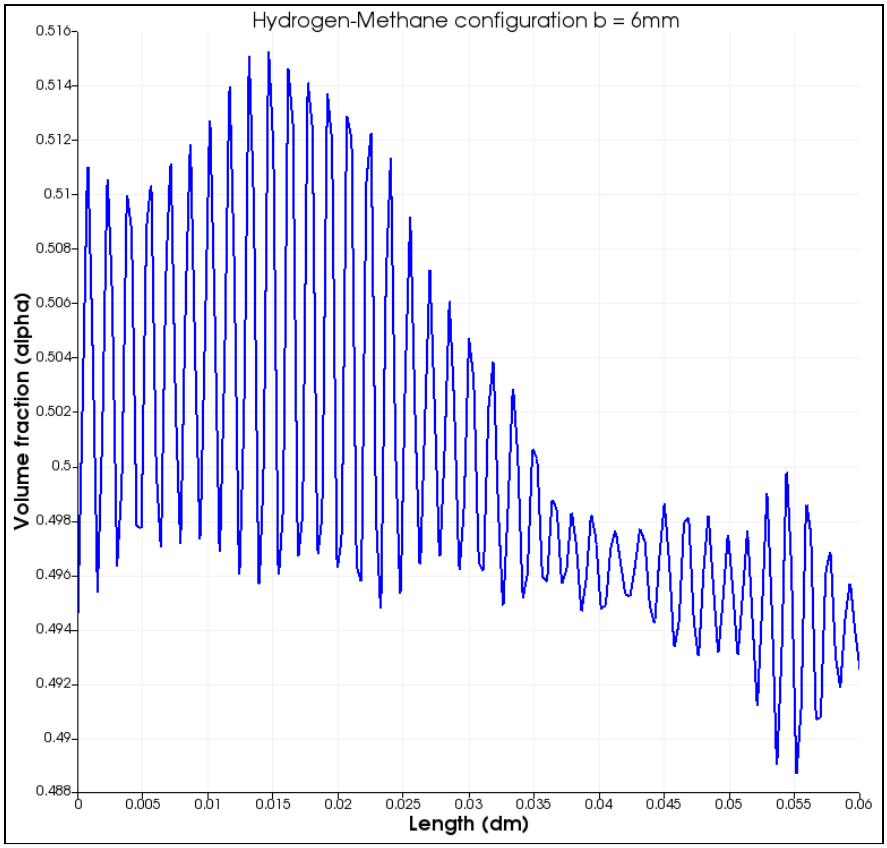
- The first two pairs hydrogen-methane and helium-hydrogen have similar density difference ($\Delta\rho \sim 10^{-5}$ g/cm^3) and average viscosity. For a Hele-Shaw geometry with height b = 6mm no change in volume fraction profiles was observed even after 10 seconds. This case was simulated for b = 20mm and 10mm. 10mm case showed some oscillations when velocity profiles were checked but the 20mm box clearly had a disturbance in the interface leading to instability.
- For the oxygen-nitrogen pair ($\Delta\rho \sim 10^{-4}$ g/cm^3), simulations were performed for 5mm, 3mm, and 1mm for 4 seconds (Less time due to system specs. constraints). Although the interface had some oscillations but the top-bottom configuration was maintained at all times for all the above heights. Thus the critical height must lie in this ballpark.
- No successful simulation i.e. without the total destruction of the top-bottom configuration could be performed for the air-helium pair ($\Delta\rho \sim 10^{-3}$ g/cm^3). The lowest value of height simulated was 0.5mm (this could also not be completed because of high computational

time) for 2 seconds. The interface separating the fluids was completely annihilated after 1.8 seconds.

- Further simulations could not be performed as per the plan due to high computational time and the reasons mentioned above.

(a) t = 0s for b = 6mm

(b) t = 10s for 6mm

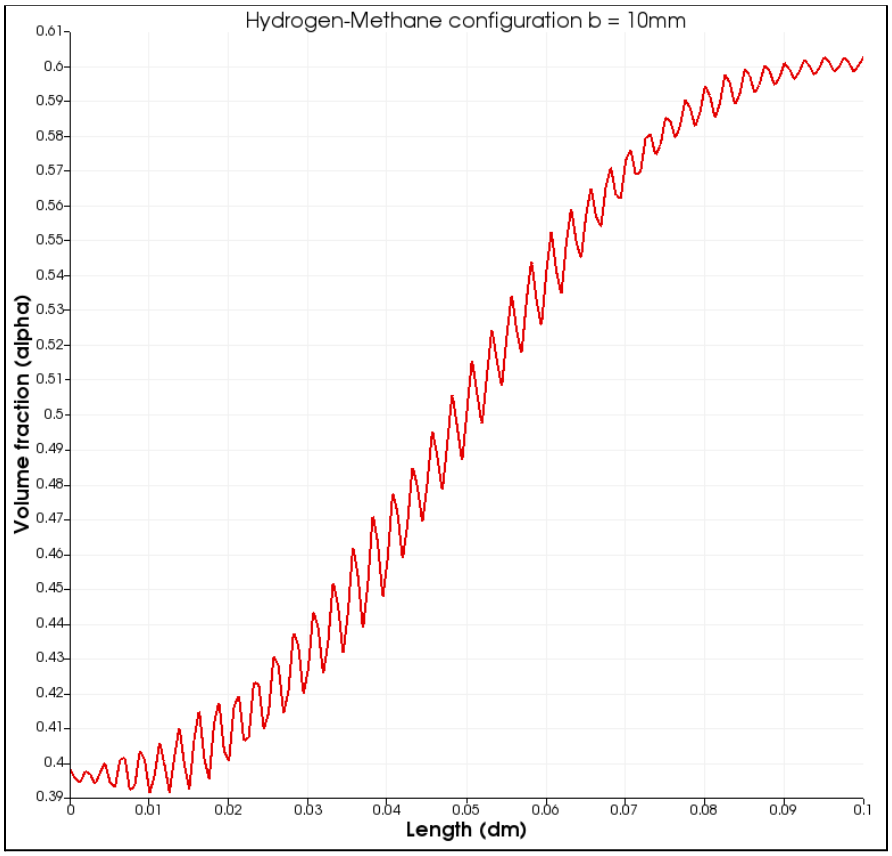


(c) Graph of volume fraction of hydrogen along the length of box in hydrogen-methane fluid configuration. It is clear that $\alpha \simeq 0.5$ which implies that even after 10s there is very little disturbance in the interface. Reducing the box height further will reduce the disturbance more but b = 6mm is a height that stops the RT instability to a very good extent.

Fig. 4 Initial and final volume fraction profiles for simulation of 10s with box height b = 6mm

(a) t = 0s for b = 10mm

(b) t = 10s for b = 10mm



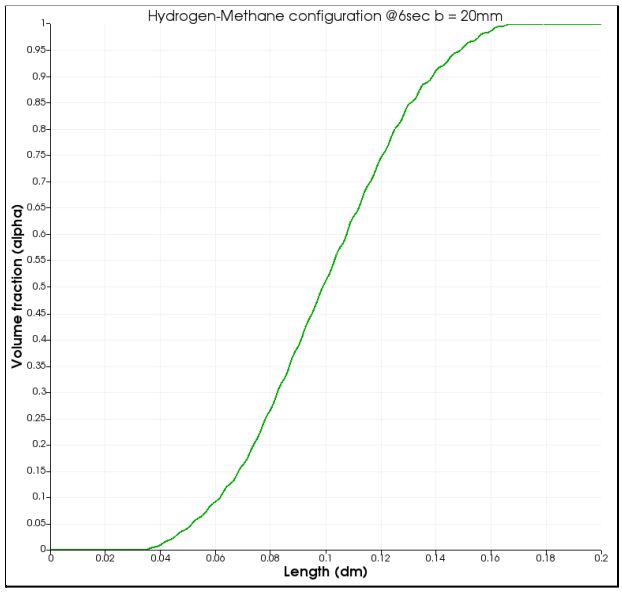
(c) Graph of volume fraction of hydrogen along the length of box in hydrogen-methane fluid configuration. It is clear that α varies from 0.5 significantly which implies after 10s there is some disturbance in the interface and the profile may lead to instability at a later time. It should be noted that although there is disturbance but the top-bottom fluid configuration is not completely destroyed. Simulation for larger times may help with a better insight in this case.

Fig. 5 Initial and final volume fraction profiles for simulation of 10s with box height b = 10mm

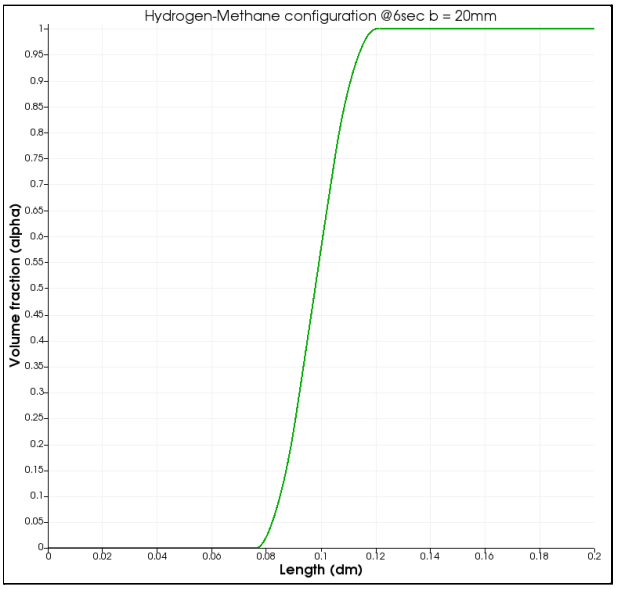
(a) t = 0s for 20mm

(b) t = 6s for 20mm

(c) t = 8s for 20mm



(d)



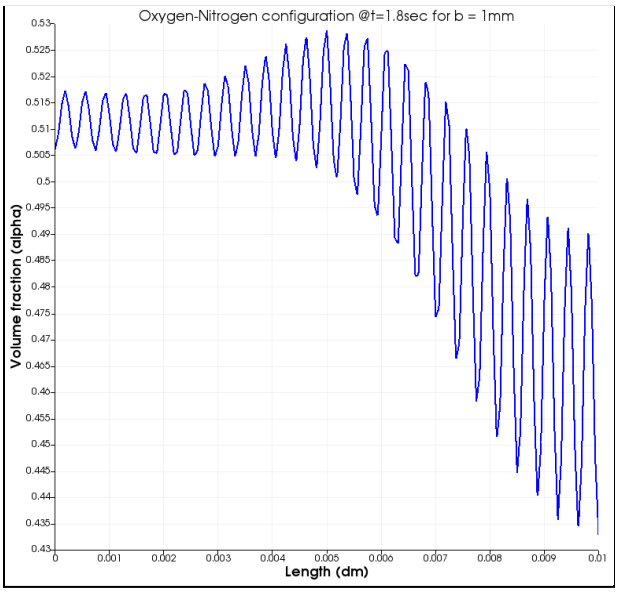
(e)

Fig. 6 Volume fraction profiles for (a) t = 0s, (b) t = 6s, (c) t = 8s with box height b = 10mm. Plots (d) and (e) show the volume fraction of hydrogen along the length of box in the hydrogen-methane fluid configuration. It is clear that α is not equal to 0.5 at all from the profiles (c) and (d). This disturbance in the interface leads to instability.

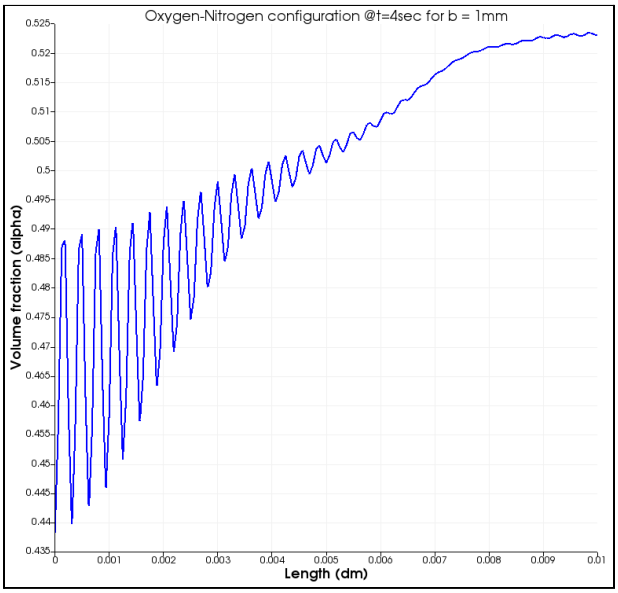
(a) t = 0s for b = 5mm

(b) t = 2s for b = 5mm

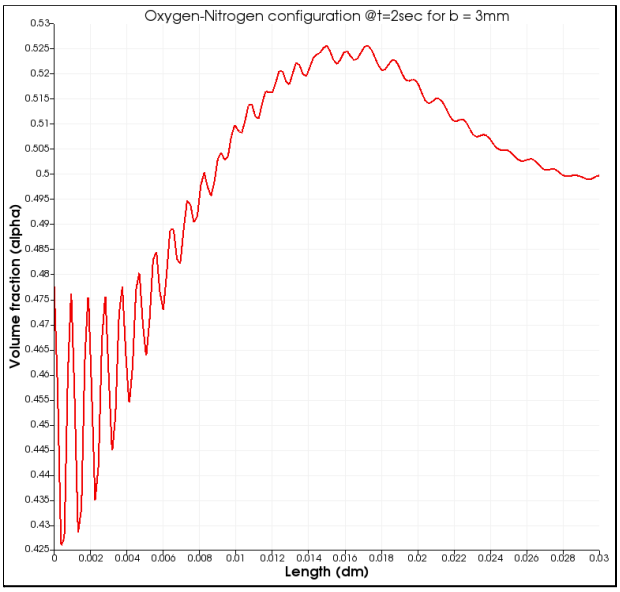
(c) t = 4s for b = 5mm



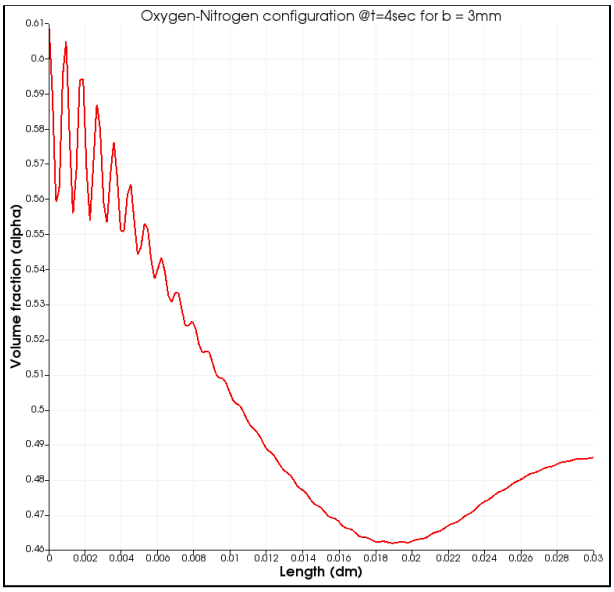
(d)



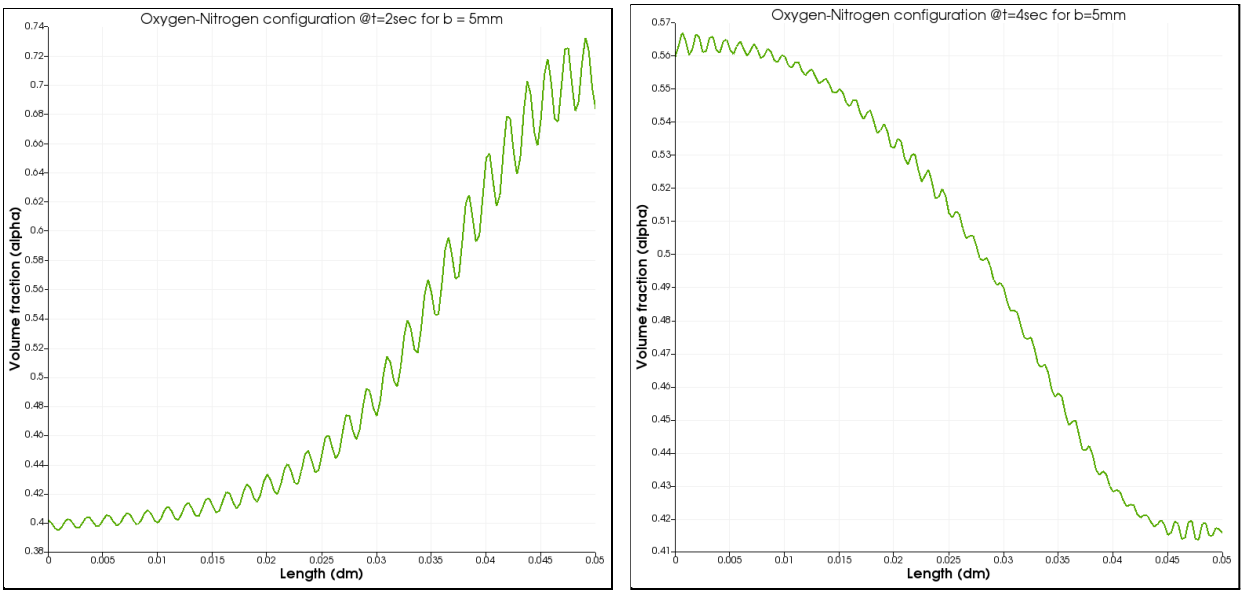
(e)



(f)



(g)



(h) (i)

Fig. 7 Volume fraction profiles for (a) t = 0s, (b) t = 2s, (c) t = 4s with box height b = 5mm. For other box heights it is more or less the same. Plots (d), (e), (f), (g), (h), (i) show the volume fraction of oxygen along the length of box in the oxygen-nitrogen fluid configuration for t = 2sec and t = 4sec with b = 1mm, 3mm, and 5mm respectively in pairwise order. It is clear that α is not equal to 0.5 as there is continuous oscillation between different time steps. For all box heights the profile at t = 2sec is tilted in opposite direction than for t = 4sec. As the box height varies we see the amplitude of oscillation from α = 0.5 varies and is larger for b = 5mm case than 1mm. As reducing the box height further meant higher computational time smaller box heights were not simulated but we can see the decrease in oscillation with heights and it is not much for 1mm. Therefore we can say that it will be slightly lesser than 1mm (~0.5mm).

5. References

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