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Plaque Concentration Modelling and Hemo-Acoustic Study in Axisymmetric Stenosed Coronary Artery

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Synopsis

The role of accurate Hemo-dynamic parameter prediction through computational simulations plays an important role in atherosclerotic surgery planning. Early diagnosis of constrictions in arteries through a cost-effective strategy can reduce the sudden deaths due to stroke. The proposed work intends to address above challenges by modelling pulsatile blood flow in constricted coronary arteries using a new solver 'LDLFoam' developed using a base template of 'icoFoam'. The validation of 2D axis-symmetric blood flow model is first carried out using the results of 3D DNS velocity profiles reported by Varghese et al. [2]. The low density lipo-protein (LDL) concentration is calculated by solving a transport equation along with Navier-Stokes equations for blood. After successful validation of N-S model, the transport model is incorporated to study LDL concentration. This model is validated with 3D numerical results presented by Fazli et al. [1]. The correlation between Wall Shear Stress (WSS) and plaque concentration may give a qualitative picture on regions susceptible for growth of further plaque (LDL). The second aspect involves calculation of an engineering parameter using lumen pressure which can indicate murmur/bruits heard on stethoscope for a diseased patient. The pulsatile blood flow in stenosed coronary artery is studied for stenosis of 50, 60, 70 % (by diameter) and L = 1D, 2D, 3D, 4D to analyse LDL concentration variation and acoustic indicator. It is observed that regions where WSS suddenly drops and goes into a small negative zone are more susceptible for plaque growth. Also, as the length of stenosis increases, larger region is found to be more susceptible for plaque growth.

keywords: *Computational Hemo-dynamics, Low Density Lipoprotiens (LDL), Sound spectrum.*

1 Introduction

Atherosclerosis is an arterial abnormality involving constriction of artery due to deposition of fatty material (called 'plaque') which mainly consists of Low Density Lipoproteins (LDL). Computational modeling of LDL growth plays an important role in surgery planning for atherosclerotic

patients. The current study involves development of OpenFOAM-based solver 'LDLFoam' which can accurately model LDL concentration. The solver is validated by comparing the LDL concentration on the top wall of un-constricted Common Carotid Artery (CCA) with steady blood-flow at inlet. The values of LDL concentration are compared with those reported in 3D numerical study of Fazli et al.[1]. Also, an axisymmetric blood-flow analysis in constricted artery with inlet Reynolds number (Re) of 500 is performed. The results for post-stenotic velocity profiles are compared with those reported in 3D DNS study of Varghese et al.[2]. A close agreement with less than 5% deviation is observed for both the cases. After validating the newly developed solver, an investigation on pulsatile blood flow in stenosed coronary artery is performed. The effect of stenosis length and severity on the hemodynamic parameter Wall Shear Stress (WSS) and LDL wall concentration is studied. The regions susceptible to higher LDL growth are identified along with corresponding WSS values so as to aid medical practitioners in deciding region for stent insertion. Also, characteristic break frequency of wall pressure rate spectrum is obtained for various configurations to get a qualitative picture of acoustic signals useful for diagnostic purpose. The mathematical model along with coupling strategy are provided in Section 2. The two validation cases used to benchmark the solver along with corresponding boundary conditions are provided in Section 3. The computational setup for pulsatile blood flow in axisymmetrically stenosed is presented in Section 4.1 whereas the results are discussed in Section 4.2.

2 Governing Equations and Models

The flow of blood in arteries can be described using the incompressible continuity and momentum equations, given as follows,

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\,\mathbf{V} = -\frac{\nabla P}{\rho} + \frac{1}{\rho}\nabla.\boldsymbol{\tau} \tag{2}$$

where '**V**' is the velocity vector of the fluid, P is the fluid pressure, ρ is the density of blood and '$\boldsymbol{\tau}$' is the viscous shear stress tensor which, in the case of Newtonian assumption is defined as:

$$\boldsymbol{\tau} = \mu\left(\nabla\mathbf{V} + \nabla\mathbf{V}^T\right) \tag{3}$$

where μ is the dynamic viscosity of blood, taken as constant in current study. In the above equation, body force terms are completely ignored assuming artery to be supine position.

As the LDL particle size are very small and their density is close to that of the blood density, the particles are considered to be passive tracers in the present work. Therefore, the LDL particles are assumed to follow the blood without imparting any inertial effects of their own and passively undergo convection and diffusion. The transport of LDL molecules in the lumen is governed by the scalar transport equation (formulated by Fatouraee et al.), given as follows:

$$\frac{\partial C}{\partial t} + (\mathbf{V}.\nabla)\,C = D_c\nabla^2 C \tag{4}$$

where C is the concentration of LDL and D_c is the diffusion coefficient and in the present work its value is taken as per medical literature. The incompressible Navier-Stokes equations (Eq. 1 and 2) are solved using SIMPLE algorithm with second order accurate Finite Volume discretisation in LDLFoam. After obtaining final flow velocities at given time-step, scalar transport model (Eq. 4) is solved implicitly using the latest velocities to obtain concentration 'C' of LDL particles. Thus, a one-way coupling is achieved between flow and concentration solver.

3 Validation of LDLFoam

In order to solve the conservation laws for blood and LDL, a coupled module is developed in OpenFOAM using the base template of icoFOAM (Laminar, Transient and incompressible flow). The scalar transport equation is added as an extra equation that needs to be solved implicitly in the current study. In order to successfully solve the scalar transport, a new flow-field variable along with corresponding transport properties are created. The newly developed solver named as 'LDLFoam' is validated for two cases involving LDL concentration study in straight Common Carotid artery and pure blood flow in constricted artery.

3.1 Case 1: Flow-LDL investigation in Straight Common Carotid Artery

LDLFoam solver is first developed and validated with benchmark problem proposed by of Fazli et al. [1]. The problem involves steady blood flow through Common Carotid Artery (CCA) of Diameter 'D' with Reynolds number (Re = ρUD/μ) of 250 and Schmidt Number (Sc = 1/(Re.D_c)) of 1.6 $\times$ 10^5 . The Computational domain along with boundary regions is presented in Figure 1. The problem is Non-dimensionalised with 'Re' and Boundary Conditions are as follows

1. Inlet: Fully developed parabolic velocity of a tube (u(r) = 2u_{mean}[1-(r/R)^2]), dP/dx = 0 and C = 1 (constant).
2. No Slip Wall: u = dp/dn = 0, v = V_w and dC/dn = $C_w V_w/D_c$ (penetration velocity V_w = 3.36 $\times$ 10^{-7})
3. Outlet : du/dx = dv/dx = P = 0 and dC/dx = 0
4. Axisymmetry : v = du/dn = dP/dn = 0 and dC/dn = 0

A length of 10D is considered in axial direction so as to compare solutions with literature. The solutions are obtained with a non-uniform mesh consisting 200 cells in axial and 100 cells in radial direction. A constant time step size of 10^{-4} is used in the current study. The Steady flow results are obtained through LDLFoam when residuals reach below certain minimum and remain constant. These results are extracted on the No-Slip wall and values for concentration 'C' are compared with those reported in 3D numerical study of Fazli et al.[1]. It can be observed from Figure 2 that a good comparison with deviations less than 5% is observed. Thus, the validity of current axisymmetric solver 'LDLFoam' for modeling LDL transport equation is established through current case study.

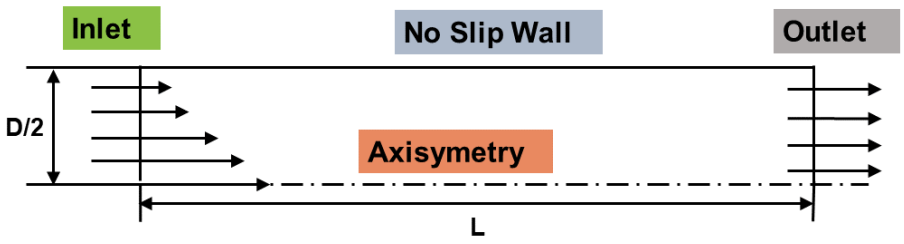


Figure 1: Computational domain for Blood Flow-LDL concentration study in Straight Common Carotid Artery (CCA).

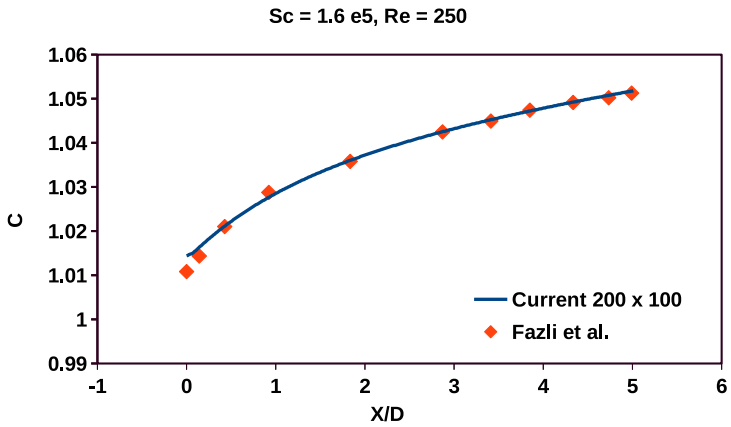


Figure 2: Comparison of LDL concentration 'C' on the top surface of artery (No Slip Wall) with values reported in 3D numerical study of Fazli et al.[1].

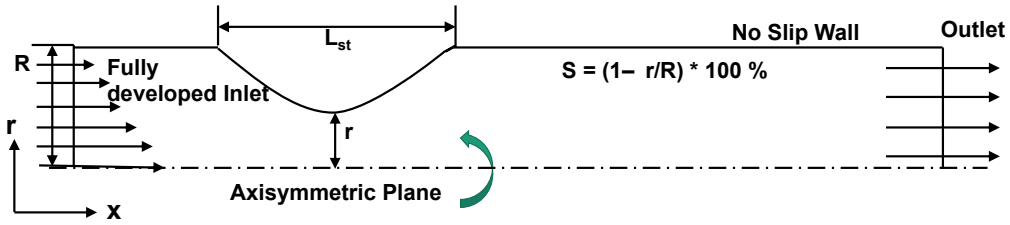


Figure 3: Computational domain for Blood Flow study in axisymmetrically constricted artery.

3.2 Case 2: Blood Flow in Axisymmetrically Constricted Artery

In order to check the suitability of current axisymmetric solver for stenosed (constricted) artery model, the flow velocities for Re = 500 and S = 75% (by area) are compared with 3D numerical solutions presented by Verghese et al. [1]. The computational domain used in the current study is presented in Figure 3. In the figure, 'x' stands for the stream-wise direction and 'r' stands for the cross-stream direction. If 'R' is the radius of the non-stenotic part, the radius of the wall of the stenosis along the x direction, R(x), is given by

$$R(x) = R - \frac{\beta_s R\,[1 + cos(2\pi(x - x_0)/L_{st})]}{2} \tag{5}$$

where R = D/2, D is the diameter of the non-stenotic lumen, L_{st} is the length of the stenotic part, and β_s = 1 − (r/R) is the percentage reduction in diameter, i.e., severity of stenosis. It can also be related to the percentage reduction in area, α_s, as $\beta_s = 1 - \sqrt{1-\alpha_s}$.

The validation of the present 2D axisymmetric model is done with the results of Varghese et al. [2]. The boundary conditions used in current study are same as those used in first validation case. Varghese et al. [2] performed 3D DNS simulation of steady flow through an axisymmetrically stenosed lumen at Re = 500 with L_{st} = 2D. The percentage reduction in area of the lumen at the stenosis throat was taken as α_s = 75%, which corresponds to β_s = 50%. The length of the upstream and downstream sections of the lumen for this case were taken as L_1 = 3D and L_2 = 15D, respectively. The Reynolds number is defined based on the average value of the inlet parabolic velocity profile, u_m, non-stenotic diameter, D, and kinematic viscosity, ν, as Re = u_mD/ν.

The Non-dimensionalised simulation was carried out with a constant time-step size of 10^{-4} so that CFL remains below 1. A multi-block meshing strategy as explained in next section is used to generate smooth mesh throughout the domain. A total of 4 blocks were used and 'polyLine' feature is employed for cosine curve generation in geometry. The comparison of axial velocity at a post-stenotic length of 4D are compared with those reported by Varghese et al.[2] and presented in Figure 4. It can be observed that for a very fine grid, the results match with 3D DNS solutions with deviation less than 5%. Thus, the validity of current OpenFOAM model to handle complex geometry is also established.

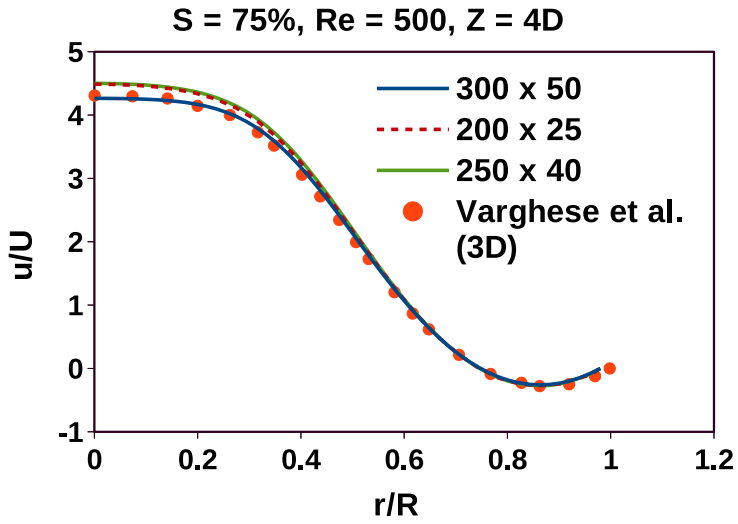


Figure 4: Comparison of axial velocity along cross-section at a post-stenotic distance of 4D with values reported in 3D DNS study of Varghese et al.[2].

4 Pulsatile Blood Flow in Stenosed Coronary Artery

A new solver named 'LDLFoam' is developed to study coupled Flow-LDL concentration phenomenon and validated with two different cases involving simple and complex geometries. After successful validation the solver, an investigation is performed on the effect of stenosis severity and its length for pulsatile blood flow in stenosed coronary artery.

4.1 Computational Setup

Coronary artery is a primary series of artery which emanates directly from Aorta. This network of artery is found to be most susceptible for deposition of plaque since blood first enters this region after getting pumped from heart. Hence, current study involves investigation of hemodynamic and hemo-acoustic parameter for pulsatile blood flow in stenosed coronary artery. The geometry used in current study is similar to the one used for second validation case (3.2) except of dimensions of geometry which are provided in Table 1. The details of parameter used in the study are also listed in the same table. The boundary conditions for current study are same as second validation case except for inlet velocity. Since the actual blood flow is pulsatile in nature with systolic and diastolic phase, a time varying parabolic velocity is specified at the inlet as follows :

$$U(t) = \begin{cases} U_{min} + \frac{U_{max}}{2}(1 - cos(2\pi t/T)) & \text{if } t/T \leq 0.5 \\ U_{min} & \text{otherwise} \end{cases} \quad (6)$$

The variation of centerline velocity at inlet along time is presented in Figure 5. The other flow parameters used for study are presented in Table 2. The Womersley number (Wo) indicates the frequency of flow pulsation and is related to flow parameters as follows :

Geometric Parameter	Value
Radius (R)	1 cm
Stenosis Severity (S)	50, 60 and 70%
Length of Stenosis (L_{st})	2R, 4R, 6R and 8R

Table 1: Details of geometric parameters studied

Flow Parameter	Value
Reynolds Number (Re = ρUD/μ)	100 – 1000 (min. – max.)
Womersley Number (Wo)	14.4 (13.8 for Aorta)
Kinematic Viscosity (ν)	3.5 $\times$ 10^{-6} m^2/s
Diffusivity (D_c)	2 $\times$ 10^{-11} m^2/s

Table 2: Flow parameters

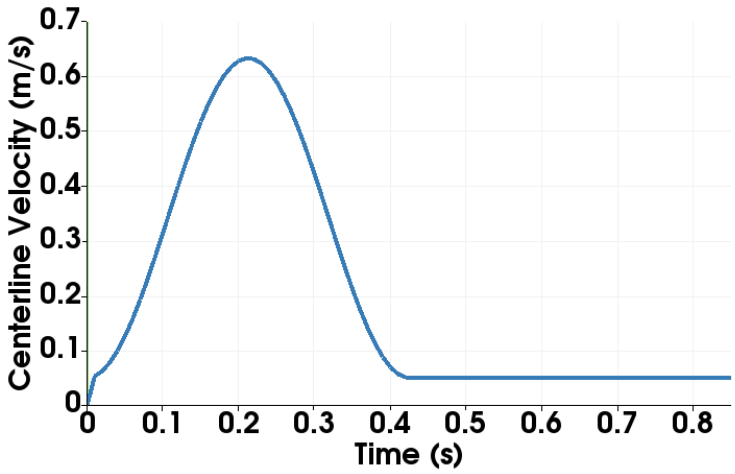


Figure 5: Centerline Inlet velocity variation with time for stenosed coronary artery investigation.

$$Wo = D\sqrt{\frac{\omega\rho}{\mu}} \tag{7}$$

where ω represents angular frequency of pulsation, ρ represents fluid density and μ denotes dynamic viscosity. The mesh was generated using 4 partitions (blocks) in the geometry as shown in Figure 6. The curved geometry is created using 'polyLine' feature in OpenFOAM and meshing is performed in multi-block method with simple grid stretching and compression. Along radial direction, a constant mesh expansion of 5 is used to resolve near- wall concentration gradient accurately. A sample mesh for S = 50% and L_{st} = 2D is shown in Figure 7 at 50% resolution. A constant time-step size of 10^{-5}s is used to limit the CFL below 2. The results are presented after 6 cycles of flow so that periodic steady-state is ensured.

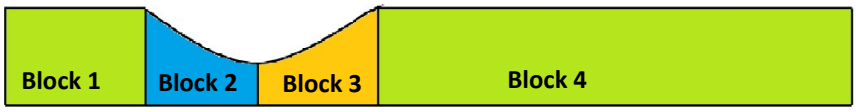


Figure 6: Multi-Block meshing strategy used for curved geometries.

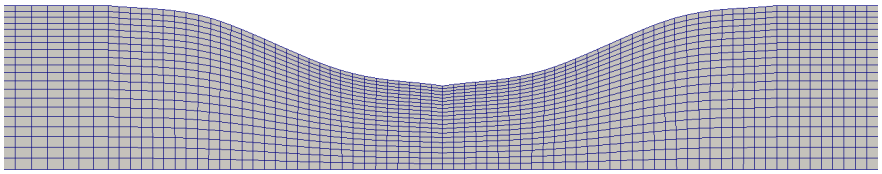


Figure 7: Sample mesh for S = 50%, L = 2D (presented at $1/2^{th}$ resolution in radial direction).

4.2 Results and Discussions

The main objective of the current study is to investigate the effect of stenosis severity and length on the phenomenon of LDL growth at arterial wall. Also, suitability of WSS as a hemodynamic parameter to study further plaque growth needs to be analyzed. Apart from LDL growth modeling, the second objective is to understand features affecting abnormal acoustic signals arising from stenosed artery. In order to understand these two processes, correlation between LDL and WSS and variation of characteristic cut-off frequency is presented.

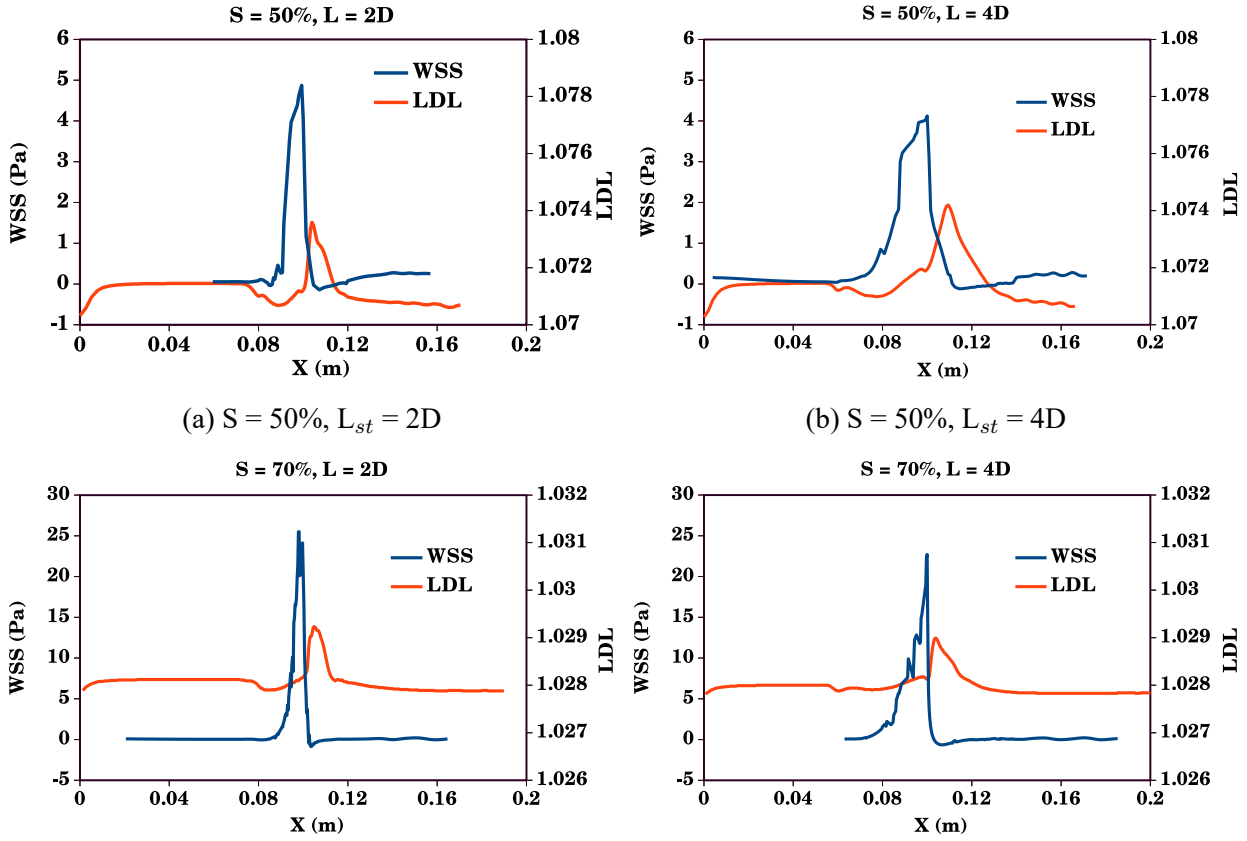
4.2.1 Wall Shear Stress (WSS) and LDL Concentration

WSS is an important parameter used in biomedical literature to understand flow behavior and identify regions of further plaque growth. The value of WSS (τ_w) is calculated from following formulation

$$\tau_w(x) = -\mu \left[\left(\frac{\partial v}{\partial r} - \frac{\partial u}{\partial x} \right) sin(2\alpha_x) + \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right) cos(2\alpha_x) \right] \tag{8}$$

where ‘u’ and ‘v’ represent axial and radial velocity components respectively and α_x(x) is the local angle of the wall with respect to the x axis.

The cycle averaged values of WSS and LDL on arterial wall are obtained from ParaFOAM and are presented for four different geometries in Figure 8. It can be observed that across stenosis, WSS magnitude increases exponentially (500% increase from S = 50% to 70%) whereas across length, the area occupied by WSS peak increases. The LDL values show similar trend across stenosis length but an inverse relation along severity. This may be attributed to the observation that probability of LDL diffusion inside domain increases due to larger velocities leading to lower concentration along wall. The region of LDL spike is observed to correspond with region involving sudden drop of WSS from its peak to a small negative value. This observation is in accordance with general physiological flow literature which suggest areas of low and oscillating WSS are susceptible for more plaque growth. Thus, the current study validates the theoretical understanding by actually modeling LDL concentration through a coupled mathematical model.



(a) S = 50%, L_{st} = 2D

(b) S = 50%, L_{st} = 4D

(c) S = 70%, L_{st} = 2D

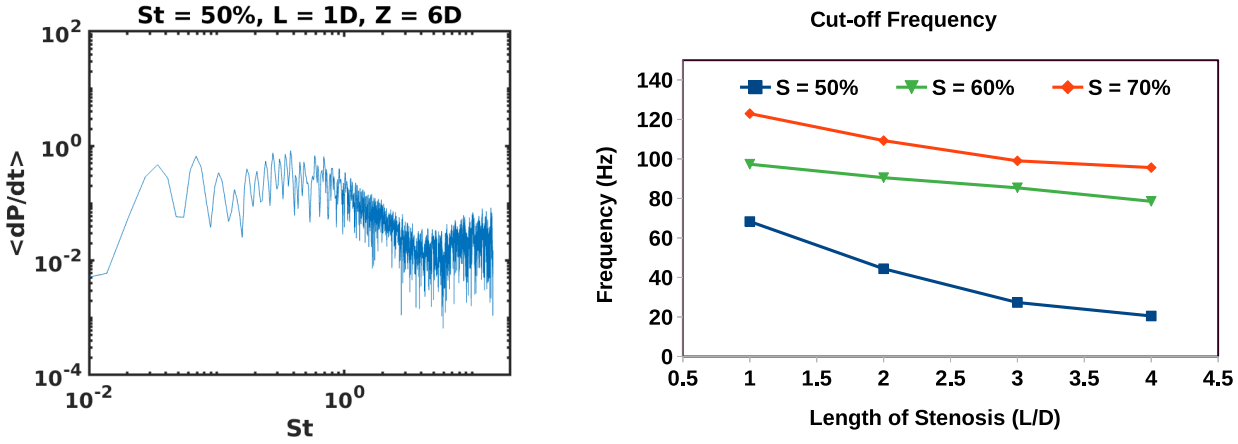
(d) S = 70%, L_{st} = 4D

Figure 8: Variation of time averaged WSS and LDL concentration along top wall for different geometric configurations.

4.2.2 Bruit Cut-off frequency

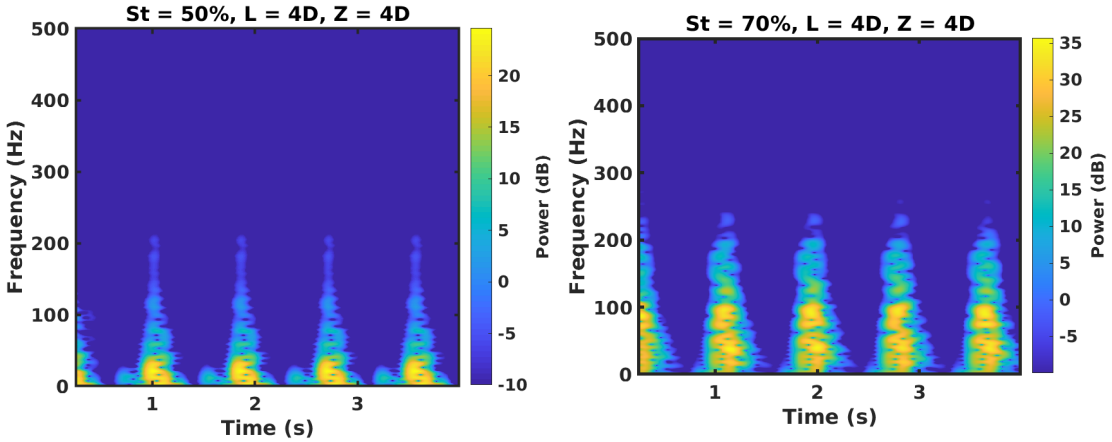
It is usually observed that a distinct murmur is heard from stethoscope for atherosclerotic patients. This distinct murmur has a characteristic cut-off frequency at which intensity of sound spectrum suddenly starts dropping. The characteristic frequency is termed as break/cut-off frequency in medical field. The current study intends to get a qualitative picture of variation of this cut-off across different geometry. For this purpose, pressure rate spectrum is considered and its cut-off is calculated as shown in Figure 9a. Since pressure fluctuations are shown to be a main source of acoustic signals in the recent Flow-Acoustic study performed by Seo et al. [3], pressure rate is considered in current study. The variation of cut-off frequency for 12 different geometric configurations are presented in Figure 9b. It can be observed that lower stenosis do not generate significant frequencies (above 100 considered significant) whereas across stenosis severity these values increase and reach significant levels. The reason may be attributed to stronger pressure fluctuation leading to higher cut-off. This hypothesis is confirmed from spectrogram investigation which provides frequency contents of signal at different time instants. A sample spectrogram for S = 50% and 75% (L_{st} = 2D) is presented in Figure 10. It can be observed that increase in stenosis severity leads to higher frequencies which stay for longer time. Thus, the current study establishes usage of break-

frequency as a suitable parameter for diagnosis of severe blockage in coronary artery.



(a) (b)

Figure 9: (a) Sample Pressure rate spectrum for S = 50%, L_{st} = 2D (b) Variation of characteristic cut-off frequency of pressure rate spectrum for various geometric configurations.



(a) S = 50%, L_{st} = 4D (b) S = 70%, L_{st} = 4D

Figure 10: Variation of time averaged WSS and LDL concentration along top wall for different geometric configurations.

5 Conclusions and Future Scope

A coupled Flow-LDL concentration solver is developed in OpenFOAM framework. The newly developed 'LDLFoam' based on icoFOAM template is validated for 2 different test cases involving LDL transport in straight un-constricted CCA and steady flow in axisymmetric stenosed artery. A good agreement with literature is obtained with deviations less than 5%. The developed solver is applied to study effect of stenosis severity and length for pulsatile blood flow in stenosed coronary

artery. Analysis of average WSS and LDL concentration values indicate regions in post-stenotic part which experience sudden drop in WSS to be more susceptible to further plaque growth. This can be of help in medical surgery planning involving stent insertion. Also, a qualitative picture of acoustic signal obtained on skin surface is examined through pressure rate spectrum on lumen wall. A characteristic cut-off frequency for spectrum is calculated for various geometric configurations. It is observed that considerable amount of frequencies are obtained for stenosis severity higher than 50% and are distinguishable in nature. Thus, the current study establishes suitability of non-invasive diagnosis of arterial abnormality through sound spectrum.

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