Report on

Mach Refection: Flow over a Forward facing Step

Case Study Project

Under the guidance of

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Submitted by

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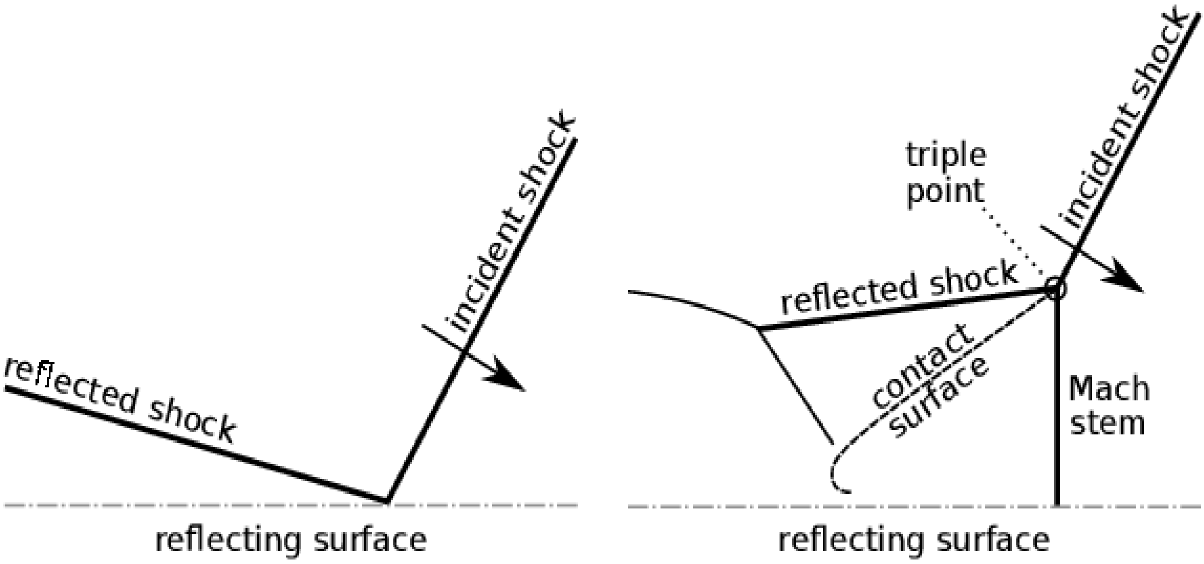
1. Introduction

A flow is termed supersonic if the flow travels faster than the speed of sound through the continuum. Speed of the sound is the speed at which any information is transmitted through the continuum. Therefore, when the flow is supersonic, there is no way the flow can know the details of what it has to encounter. If it encounters an obstacle, the flow is committed to a sudden, discontinuous change resulting in loss of speed and increase in pressure and temperature. This is idea of shockwaves. Shock waves are very thin regions in the fluid where the fluid properties change by a large amount. In many flow problems multiple shocks are present. The shocks may intersect with each other and with the surfaces generating them.



(a) (b)

Figure 1. An incident shock travelling towards a surface adopts on of the two configurations; (a) Regular reflection; (b) Mach reflection [3].

The problem of shock reflections is of importance to unsteady gas dynamics, including the study of detonation waves. When a shock wave reflects from a solid surface or a plane of symmetry, it usually adopts one of two forms: a regular reflection or a Mach reflection, illustrated in fig.1.

The Mach reflection consists of three shocks: the Mach stem, the incident shock and the reflected shock. The point where the three shocks intersect is called the triple point. The reflected shock emanates from the triple point travels transversely behind the incident shock. A slip-line or a contact surface separates the gas shocked by the Mach stem and the gas shocked by the incident and reflected shock waves.

The configuration of the three discontinuities is determined by the incident shock strength, the angle between the incident shock and the Mach stem, and the properties of the fluid. The theory of Mach reflection is discussed in detail in section 2.2.

2. Governing Equations

The Navier-Stokes equations for an inviscid, compressible flow in an arbitrary domain is

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla.\left[\vec{u}(\rho\vec{u})\right] + \nabla p = 0$$

where all symbols have their usual meaning. The Navier-Stokes equation is supplemented with the conservation of mass

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{u}) = 0$$

Conservation of total energy for an inviscid compressible flow gives

$$\frac{\partial(\rho E)}{\partial t} + \nabla.\left[\vec{u}(\rho E)\right] + \nabla.(p\vec{u}) = 0$$

where the total energy density $E = e + |\vec{u}|/2$ with e the specific internal energy.

The 3 equations are supplemented with an equation of state which is the isentropic relation

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial\rho}\right)_s = a^2$$

where a is the speed of sound.

2.1. Shock Reflection

Consider an oblique shock wave incident on a solid wall as shown in fig. 2. The boundary condition at the wall is that, the flow immediately adjacent to the wall must be parallel to the wall.

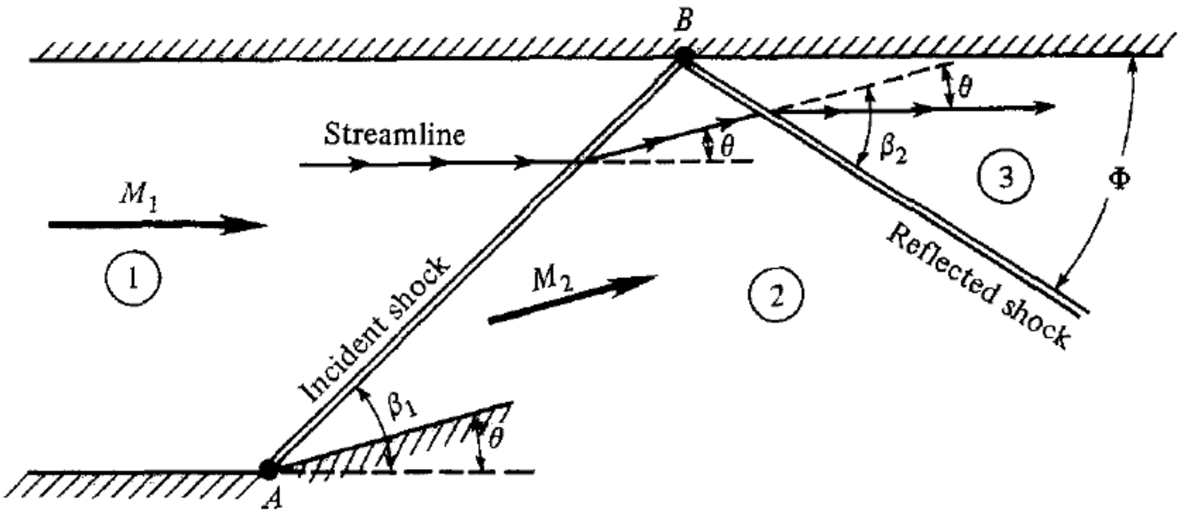


Figure 2. Regular reflection from a solid boundary [1].

The flow in region 1 with Mach number M_1 is deflected through an angle of θ at point A as shown in fig. 1. An oblique shockwave is generated at A, that impinges on the upper wall at B. In the region 2 behind the incident shockwave, the flow is inclined at an angle θ with the upper wall. The flow conditions in region 2 is defined by the oblique shock relations across the incident shock wave for Mach number M_1 and deflection angle θ. At point B, the flow should be parallel to the upper wall. Therefore, the flow will have to be deflected at angle θ downwards. This is possible only by a second shockwave, emanating from B, which is strong enough to turn the flow by an angle θ. Since $M_2 < M_1$, the reflected shock isn't as strong as the incident shock for the same flow deflection θ. This also means that the angle the reflected shock wave makes with the upper wall Φ is not equal to β_1, the angle the incident shock wave makes with the wall.

2.2. Mach Reflection

Consider a flow similar to the one in section 2.1. The discussion in section 2.1 was that the flow in region 2 of Mach M_2 is deflected by the same angle θ as the flow deflection of region 1. A reflected shockwave deflects the flow. The assumption here is that the deflection angle θ is less than the maximum achievable deflection angle for oblique shock for a flow of Mach M_2. Consider the oblique shock $\theta - \beta - M$ curves for both M_1 and M_2 as shown in fig. 3.

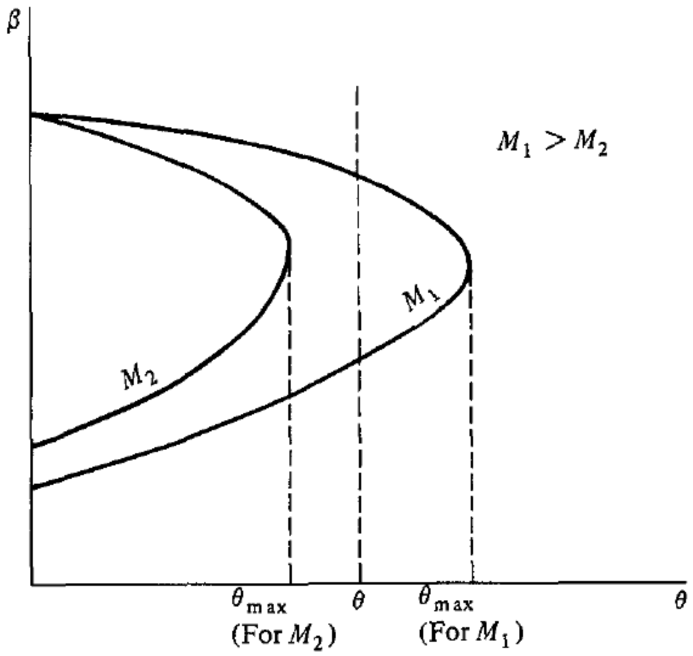


Figure 3. $\theta - \beta - M$ curves for M_1 and M_2 showing the allowable deflection angle [1].

Fig. 3 shows the situation when (θ_{max} for M_2) $< \theta <$ (θ_{max} for M_1). For the incident shockwave, the deflection angle is within the allowable deflection angle for oblique shock for M_1. Therefore, the incident shockwave is a straight line. On the other hand, the flow in region 2 cannot deflect by angle θ to satisfy the wall boundary condition with the help of an oblique shock as it exceeds the allowable deflection angle for oblique shock for a flow of Mach M_2. Instead, a normal shock is formed at the upper wall to allow the flow to remain parallel to the wall. Away from the wall, the normal shock transits into a curved shock which intersects the incident oblique shock. This interaction generates a reflected shock propagating downstream. Such a shock reflection pattern is called Mach reflection. Fig. 4 illustrates the shock pattern in Mach reflection.

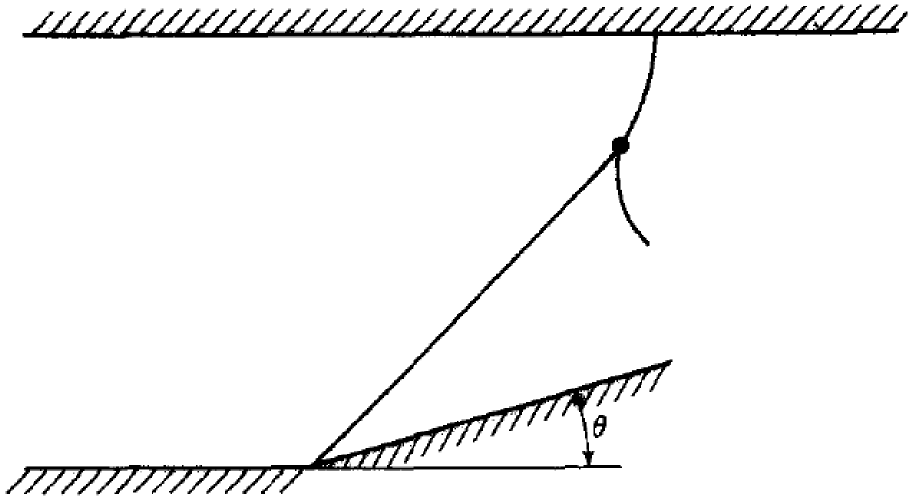


Figure 4. Mach reflection [1].

The flow downstream near the normal shockwave is subsonic and it is separated from the flow behind the curved shock by a slip-line or a contact surface as shown in fig. 1. Unlike regular reflection, Mach reflection cannot be analytically solved and requires more sophisticated numerical techniques for analysis.

3. Implementation in OpenFOAM

3.1. Problem Statement

The problem considers a supersonic flow of air ($\gamma = 1.4$) at $M = 3$ over a wedge of a forward facing step. The free stream pressure and temperature is 1 Pa and 1 K respectively. It is a classical two-dimensional test case introduced for the first time by Emery [5], and later studied by Woodward and Colella [6].

3.2. Geometry & Meshing

The geometry of the forward facing step is shown in fig. 5. The length of the domain is 3 m and the height at inlet is 1 m. A forward facing step of height 0.2 m is located at a distance of 0.6 m form the inlet. The depth (into the sheet) of the geometry is 0.1 m. The geometry was created using blockMesh utility. The meshing is simpleGrading.

The geometry is divided into 3 blocks. Each block is meshed separately, with the block near the obstacle being the most refined. No cell inflation was used.

The meshing is shown in fig. 6

Only one cell is considered along z-axis, making the simulation 2D in xy-plane.

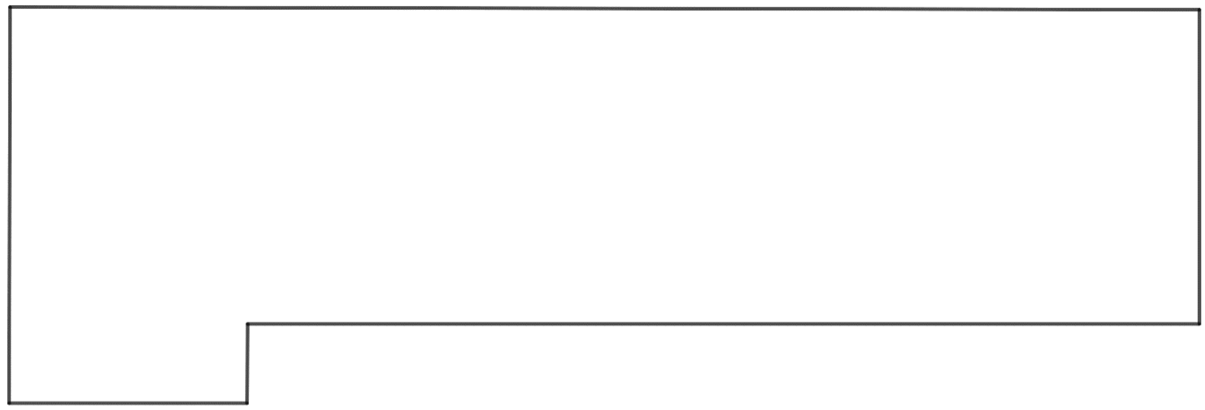


Figure 5. The configuration of flow across a forward facing step.

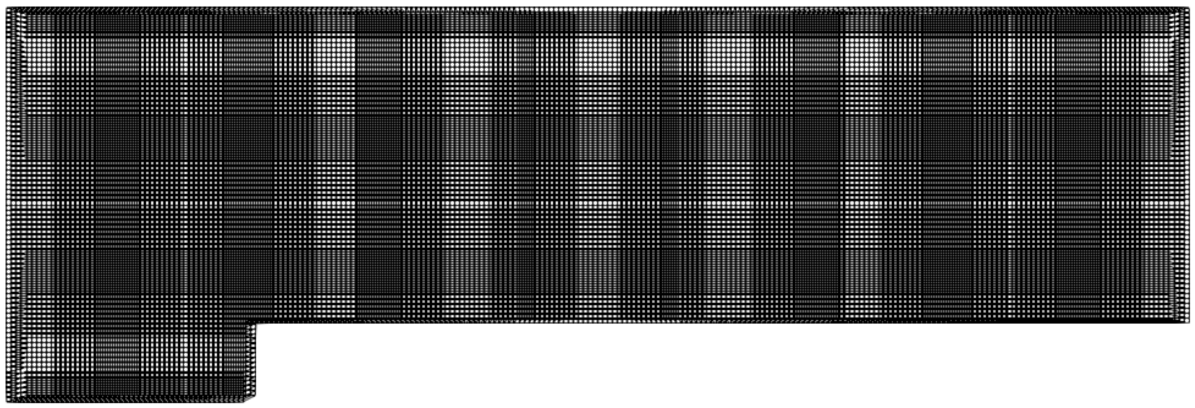


Figure 6. Meshing of forward facing step.

3.3. Initial & Boundary Conditions

The boundary conditions for various faces are described below:

a) Inlet

Pressure (p)	1 Pa
Temperature (T)	1 K
Velocity vector ($\vec{u}$)	(3, 0, 0) m/s

b) Outlet

Pressure (p)	Zero Gradient
Temperature (T)	1 K
Velocity vector ($\vec{u}$)	(3, 0, 0) m/s

c) Bottom

Pressure (p)	Symmetry Plane
Temperature (T)	Symmetry Plane
Velocity vector ($\vec{u}$)	Symmetry Plane

d) Obstacle: The base of block 2 and block 3

Pressure (p)	Zero Gradient
Temperature (T)	Zero Gradient
Velocity vector ($\vec{u}$)	Slip

e) Top: The upper face of all 3 blocks

Pressure (p)	Symmetry Plane
Temperature (T)	Symmetry Plane
Velocity vector ($\vec{u}$)	Symmetry Plane

For initial condition, the internal field is assigned Inlet boundary condition throughout the domain.

3.4. Solver

The flow through convergent-divergent nozzle governing equations, as described in section 2, are solved using rhoCentralFoam [4]. The thermophysical properties of air, assuming perfect gas, is used. The simulation type is laminar.

4. Results

The simulations are run on OpenFOAM 5.0 and the post processing is done using ParaView.

The velocity field at 4 different computational time T_{comp} until reaching steady state is shown in fig. 7a-7d and fig. 8a-8d respectively. The Mach reflection is clearly visible in the contours.

As clearly indicated in the contours, there is a sudden drop in velocity and rise in pressure across the shock.

The steady-state velocity and pressure field are shown in fig. 7d and fig. 8d respectively. The steady-state contours clearly show the Mach reflection near the upper wall. The presence of normal shock near the wall is verified using the pressure plot along x-axis.

The pressure plot along x-axis at three different y locations (heights) are shown in fig. 9a-9c.

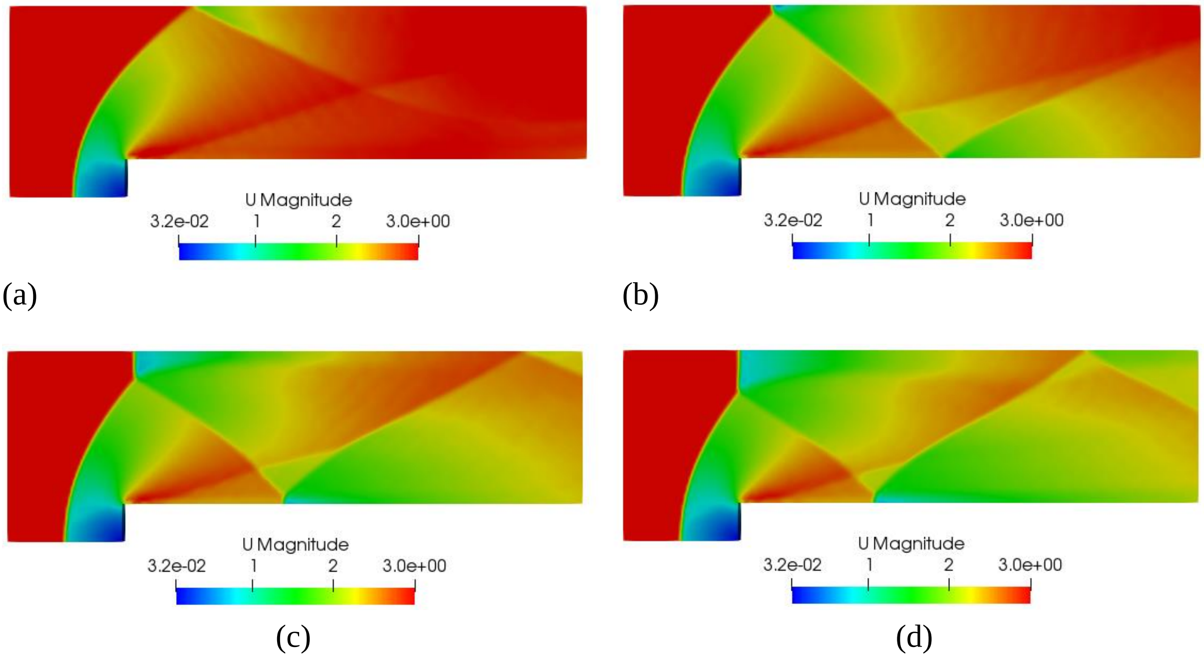


Figure 7. Velocity magnitude at (a) $T_{comp} = 1$; (b) $T_{comp} = 2$; (c) $T_{comp} = 3$ and; (d) $T_{comp} = 4$.

As indicated in the steady-state contours, the flow from inlet encounters the step and forms a bow shock in front of it. Fig. $7d$ shows the presence of a stagnation point in front of the step. The bow shock weakens away from the step and becomes an oblique shock. This is the incident shockwave for the Mach reflection at the upper wall.

The reflected shockwave acts as the incident shockwave for a second regular reflection at the lower wall. The reflected shockwave from the regular reflection at the lower wall is again reflected regularly at the upper wall. Therefore, the simulation generates one Mach reflection and two regular reflections.

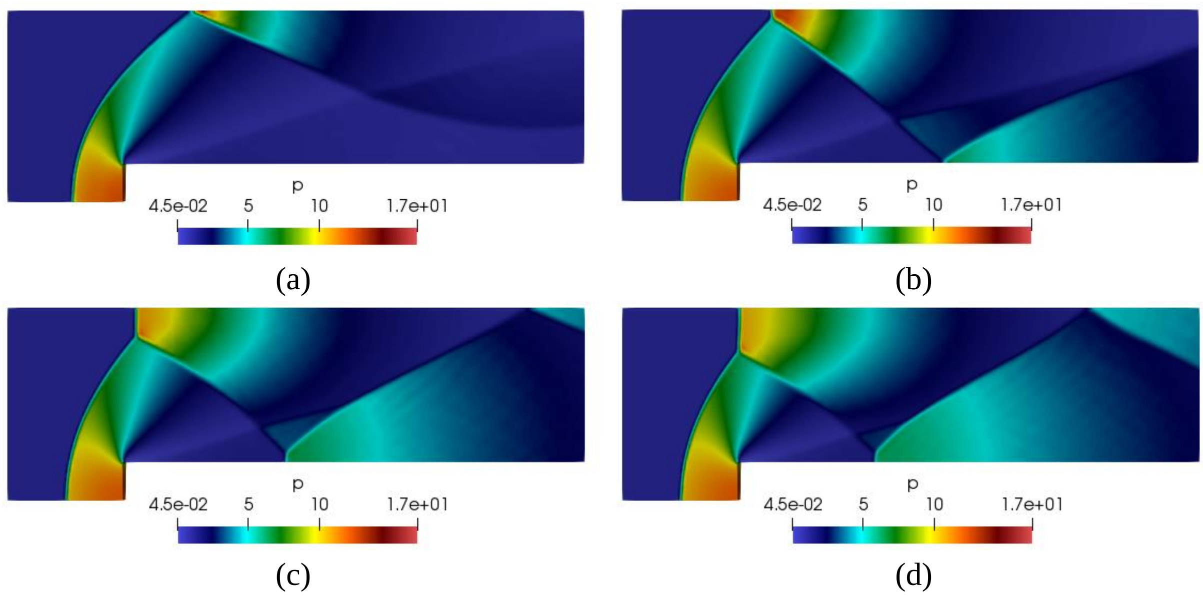
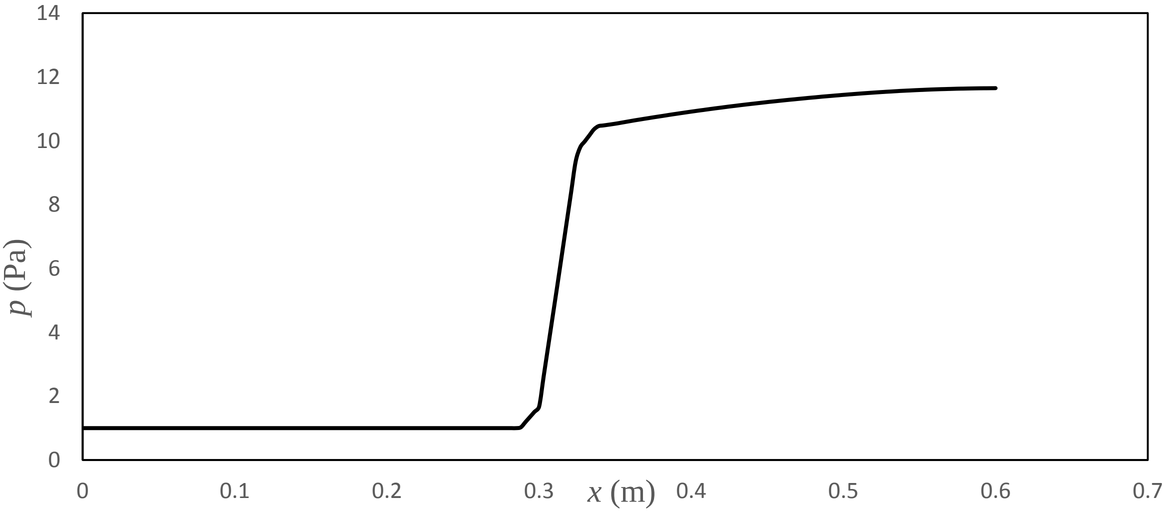
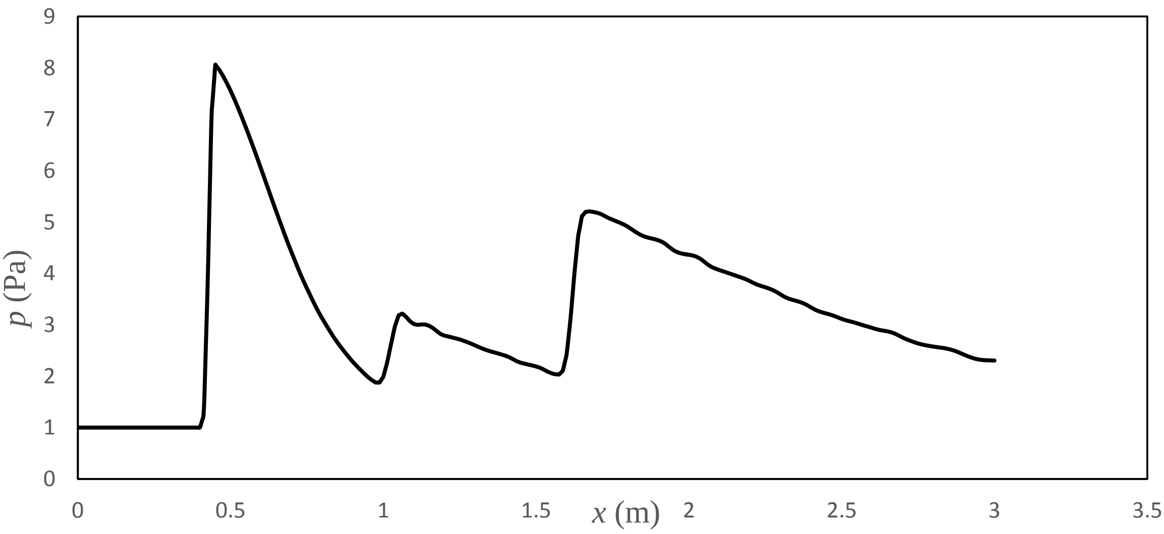


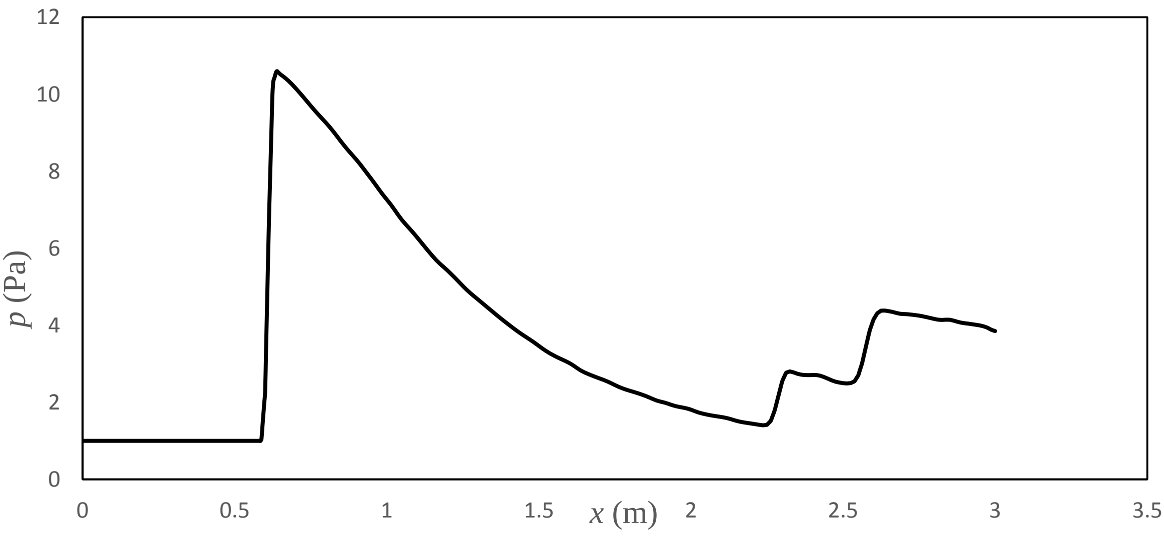
Figure 8. Pressure field at (a) $T_{comp} = 1$; (b) $T_{comp} = 2$; (c) $T_{comp} = 3$ and; (d) $T_{comp} = 4$.



(a)



(b)



(c)

Figure 9. Variation of steady-state pressure along x-axis at
(a) $y = 0.1$ m; (b) $y = 0.5$ m; (c) $y = 0.9$ m.

In fig. $9a$, the bow shock in front of the step is analysed. It can be seen that the pressure ratio across the shock p_2/p_1 is 10.46. The normal shock relation [2] for $M = 3$ shows that the pressure ratio p_2/p_1 is 10.33, indicating that the bow shock acts like a normal shock near the step. The bow shock stand-off distance δ is 0.315 m.

In fig. $9b$, all the curved, oblique shocks are analysed. There are three shocks that intersect the $y = 0.5$ plane. The pressure ratio p_2/p_1 across the three shocks are 8.06, 1.683 and 2.56 respectively. The incident shockwave of Mach reflection at the upper wall is stronger than the reflected wave. On the other hand, the incident shockwave of regular reflection at the lower wall is weaker than the reflected wave.

In fig. $9c$, the shocks near the upper wall are analysed. There are three shocks that intersects the $y = 0.9$ plane: the Mach stem and the incident and reflected shock wave of the regular reflection at the upper wall. The pressure ratio p_2/p_1 across the Mach stem is 10.6, which is close to the $p_2/p_1 = 10.33$ obtained from the normal shock relation for $M = 3$. The pressure ratio across the incident and reflected shock of the regular reflection at the upper wall are 2 and 1.76 respectively. The result shows that the reflected shock is weaker than the incident shock.

The streamlines at steady state is shown in fig. 10.

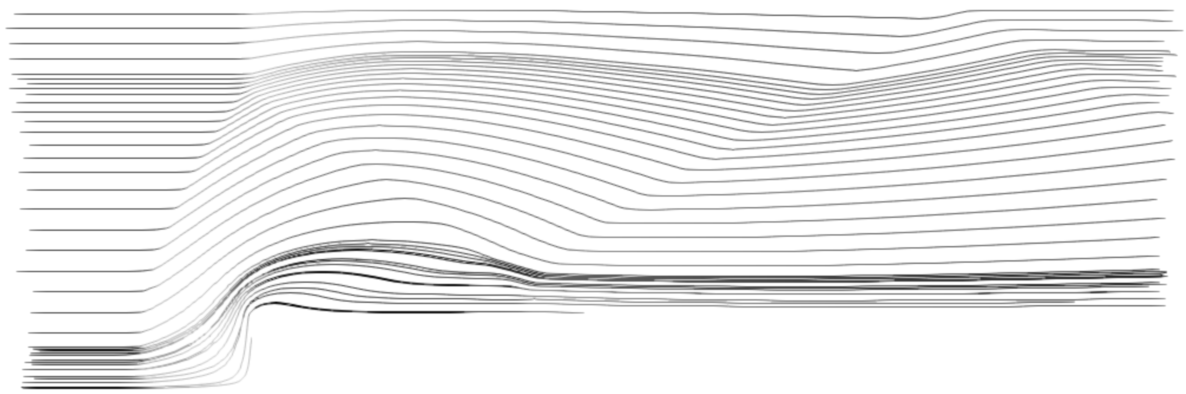


Figure 10. Steady-state streamlines.

Fig. 10 shows that the flow near the walls are parallel to walls. Also, the formation of slip-lines can be seen away from the wall at the region of Mach reflection.

5. Conclusion

Supersonic flow of air across a forward facing step is simulated using OpenFOAM solver rhoCentralFoam. The simulation produced expected result. A curved shock wave at a finite stand-off distance was produced in front of the step. Mach reflection and regular reflections at solid boundaries were observed. The pressure ratio across the Mach stem confirmed that they act as normal shocks. Analysis of the pressure ratios across incident and reflected shocks showed that the former is generally stronger than the latter. The simulation also indicated the case where the reflected shockwave is stronger than the incident shockwave. The streamlines plot also indicated the presence of slip-lines at the region of Mach reflection.

References

1. Anderson, J.D., Modern Compressible Flow, McGraw Hill Inc., New York, 1984.

2. Anderson, John D. Fundamentals of Aerodynamics. Boston: McGraw-Hill, 2001.

3. Lau-Chapdelaine, S.S., & Radulescu, M.I. (2016). Viscous solution of the triple-shock reflection problem. Shock Waves, 26, 551-560.

4. Greenshields, C. J., Weller, H. G., Gasparini, L. and Reese, J. M. (2010), Implementation of semi-discrete, non-staggered central schemes in a collocated, polyhedral, finite volume framework, for high-speed viscous flows. Int. J. Numer. Meth. Fluids, 63: 1-21. doi:10.1002/fld.2069.

5. A. F. Emery. An evaluation of several differencing methods for inviscid fluid flow problems. Journal of Computational Physics, 2(3):306–331, 1968.

6. P. Woodward and P. Colella. The numerical simulation of two-dimensional fluid flow with strong shocks. Journal of computational physics, 54(1):115–173, 1984.