Report on

Pressure Driven Flow through a Nozzle connected to a Reservoir

Case Study Project

Under the guidance of
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Contents

1. Introduction

A flow is termed supersonic if the flow travels faster than the speed of sound through the continuum. Speed of the sound is the speed at which any information is transmitted through the continuum. Therefore, when the flow is supersonic, there is no way the flow can know the details of what it has to encounter. If it encounters an obstacle, the flow is committed to a sudden, discontinuous change resulting in loss of speed and increase in pressure and temperature. This is idea of shockwaves. They are discontinuities that form in order for the flow to meet some downstream conditions. If a supersonic flow never encounters something downstream, like an obstacle or back-pressure, shocks will never arise. Similarly, for subsonic flows can never generate shocks as any downstream obstacles or back-pressure is communicated upstream and the flow curves around the obstacle.

2. Governing Equations

The Navier-Stokes equations for an inviscid compressible flow in an arbitrary domain is

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla.[\vec{u}(\rho\vec{u})] + \nabla p = 0$$

where all symbols have their usual meaning. The Navier-Stokes equation is supplemented with the conservation of mass

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{u}) = 0$$

Conservation of total energy for an inviscid compressible flow gives

$$\frac{\partial(\rho E)}{\partial t} + \nabla.[\vec{u}(\rho E)] + \nabla.(p\vec{u}) = 0$$

where the total energy density $E = e + |\vec{u}|/2$ with e the specific internal energy.

The 3 equations are supplemented with an equation of state which is the isentropic relation

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial\rho}\right)_s = a^2$$

where a is the speed of sound.

2.1. Quasi-One-Dimensional Flow

The flow through a variable-area duct is three-dimensional in reality. But with the quasi-one-dimensional assumption, the flow through the area-variable duct varies only as a function of x, i.e., $u = u(x)$, $p = p(x)$, etc. This assumption that flow properties are uniform across any given cross section represent values that are some kind of mean of the actual flow properties distributed over the cross section clearly shows that the quasi-one-dimensional flow is an approximation to the actual physics of the flow.

Consider an incremental volume as shown in fig. 1.

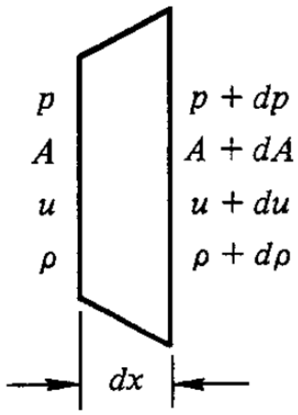

Figure 1. Incremental Volume [1].

Considering this infinitesimal control volume for conservation of mass, momentum and energy equation, and a few algebraic simplifications we get

$$d(\rho u A) = 0, \tag{1}$$

$$dp = -\rho u \, du, \tag{2}$$

$$dh + u \, du = 0 \tag{3}$$

where h is the specific enthalpy.

Equations (1) and (2) along with the isentropic relation gives

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u} \tag{4}$$

Equation (4) is called the area-velocity relation. It can be inferred from Equation (4) that for a gas to expand isentropically from subsonic to supersonic speeds, it must flow through a convergent-divergent duct.

2.1.1. Analysis of Normal Shock Waves

Consider a standing normal shock in a section of a varying area as shown in fig. 2. The control volume includes the shock wave and infinitesimal amount of fluid on either side of the shock. Since the shock wave is very thin (of the order 10^{-6} m), the control volume is extremely thin allowing for the following assumptions without introducing error in the analysis:

1. The area on both sides of the shock can be considered to be equal.
2. There is negligible surface contact with walls and thus the frictional effects can be neglected.

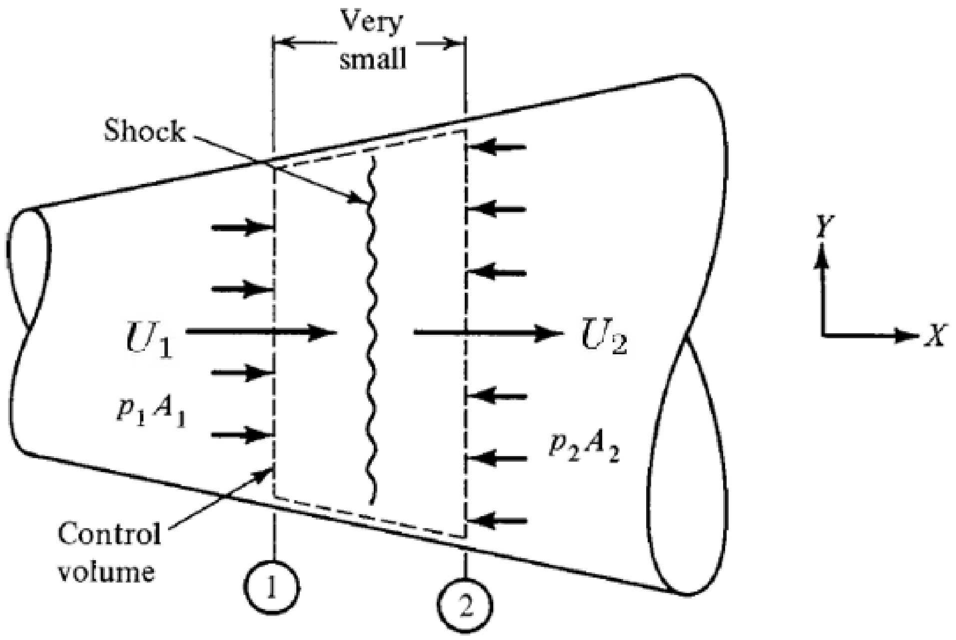

Figure 2. Control volume for shock analysis [3].

The flow is assumed to be steady, adiabatic one-dimensional flow.

The continuity equation is

$$\rho_1 U_1 = \rho_2 U_2$$

The conservation of momentum gives

$$p_1 + \rho_1 U_1^2 = p_2 + \rho_2 U_2^2$$

The conservation of energy gives

$$h_1 + \frac{U_1^2}{2} = h_2 + \frac{U_2^2}{2}$$

where all symbols have their usual meaning.

The perfect gas equation of state assuming constant specific heats gives,

$$h = \int_0^T c_p dT = \left(\frac{\gamma R}{\gamma - 1}\right) T \qquad and, \qquad p = \rho R T$$

With the above equations, the problem is fully defined. Rigorous algebraic simplification of the above equations give the normal shock relations as below.

Mach number behind the shock

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

Temperature ratio

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2}$$

Pressure ratio

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

Density and velocity ratio

$$\frac{\rho_2}{\rho_1} = \frac{U_1}{U_2} = \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2}$$

The above shock relations and the isentropic relations are used in the nozzle design and problem setup in section 3.3.

3. Implementation in OpenFOAM

3.1. Problem Statement

The problem considers steady, inviscid, non-heat-conducting flow of air through a nozzle connected to a reservoir. The pressure and temperature in the reservoir is 30 bar and 298 K respectively. Air from the reservoir is forced through a nozzle, described in section 3.2, with a back-pressure at the exit of the nozzle. Calculation of the back-pressure is described in section 3.3.

3.2. Geometry & Meshing

A nozzle is connected to the reservoir as shown in fig. 3. The height of the nozzle at the throat is half of that at the exit. The reservoir is 0.37 X 0.385 m along x and y direction respectively. The height of the nozzle at the exit is 0.03 m.

The meshing is done using OpenFOAM utility blockMesh. The geometry is divided into 8 blocks and each block has its own simpleGrading mesh. The mesh is inflated along x and y direction. Mesh is coarse in the reservoir except in the area connecting the nozzle. The mesh is refined throughout the nozzle.

Only one cell is considered alone the z direction making the simulation 2D in xy-plane.

Figure 3. The configuration of pressure flow through a nozzle connected to a reservoir.

The meshing is shown in fig. 4.

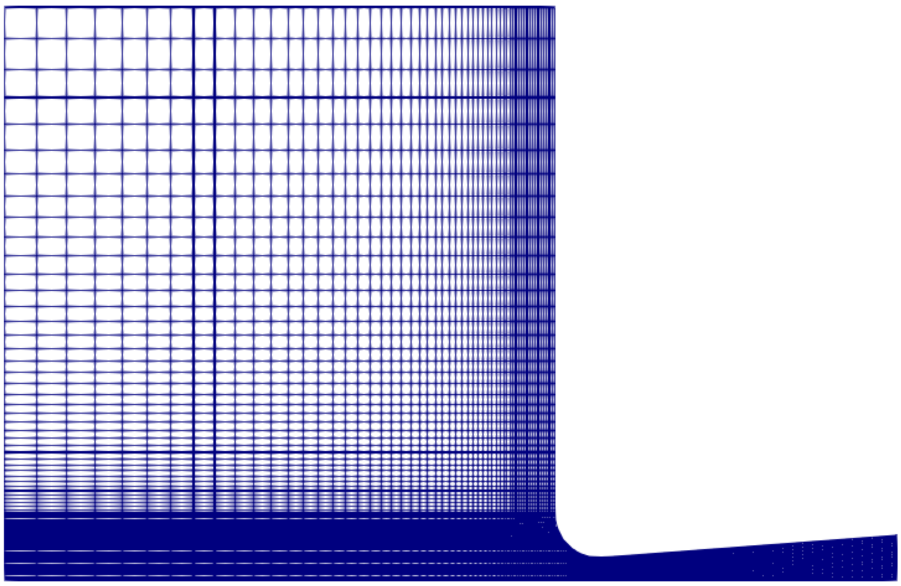

Figure 4. Meshing of the geometry.

The intense density of the mesh throughout the nozzle is clearly indicated in fig. 4.

3.3. Initial & Boundary Conditions

The boundary conditions for various faces are described below:

a) Inlet: The left face of the reservoir

Pressure (p)	30 bar = 3000000 Pa
Temperature (T)	298 K
Velocity vector ($\vec{u}$)	Zero Gradient

b) Outlet: The right face of the nozzle

Pressure (p)	p_{out}
Temperature (T)	Zero Gradient
Velocity vector ($\vec{u}$)	Zero Gradient

c) Bottom: The base of the geometry

Pressure (p)	Symmetry Plane
Temperature (T)	Symmetry Plane
Velocity vector ($\vec{u}$)	Symmetry Plane

d) Nozzle: The upper and right faces of the reservoir and the upper face of the nozzle

Pressure (p)	Zero Gradient
Temperature (T)	Zero Gradient
Velocity vector ($\vec{u}$)	Slip

The inlet pressure and temperature is assigned initially for the calculation. A velocity of 7 m/s is used for initial condition.

3.3.1. Calculation of Nozzle Exit Pressure

The aim is to generate a shock in the nozzle section. The ratio of exit area to that of the throat is

$$A_e/A_{throat} = 2$$

But the flow is sonic at the throat. Therefore, $A^* = A_{throat}$.

The shock should be generated in the nozzle such that, at the location of the shock

$$1 < \frac{A}{A^*} < 2$$

Considering $A/A^* = 1.53$, the isentropic flow relations establishes a flow of $M = 1.88$ before the shock.

Using the shock relations derived in section 2.1.1, for $M = 1.88$, $p_0 = 3 \text{ bar}$ and $T_0 = 298K$, the exit pressure is calculated as

$$p_{out} = 2.0674 \text{ MPa}$$

This is the boundary condition imposed at the nozzle exit.

3.4. Solver

The flow through a pressure driven nozzle governing equations, as described in section 2, are solved using rhoCentralFoam [2]. The thermophysical properties of air, assuming perfect gas, is used. The simulation type is laminar.

4. Results

The simulations are run on OpenFOAM 5.0 and the post processing is done using ParaView.

The steady-state pressure field is shown in fig. 5.

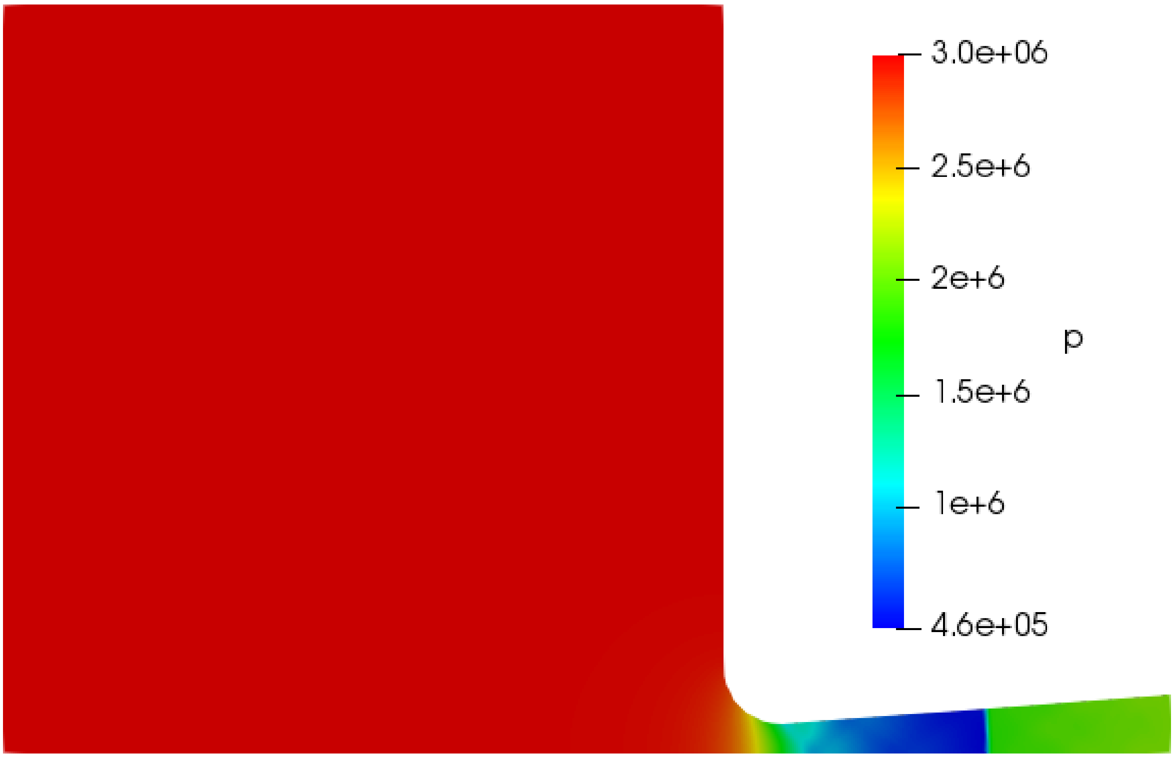

Figure 5. Steady-state pressure field.

The standing normal shock is clearly visible in the nozzle section. The contours also indicate the sudden rise in pressure across the shock wave.

The Mach number and pressure variation along the x-axis just above the bottom face is shown in fig. 6 and 7 respectively.

The plots clearly show the drop in Mach number and the rise in pressure and temperature across the shock.

The shock is located at $x = 0.504$ m. The area of the nozzle at the corresponding section is given by $A/A^* = 1.52$, which is very close to the assumed $A/A^* = 1.53$ for calculation.

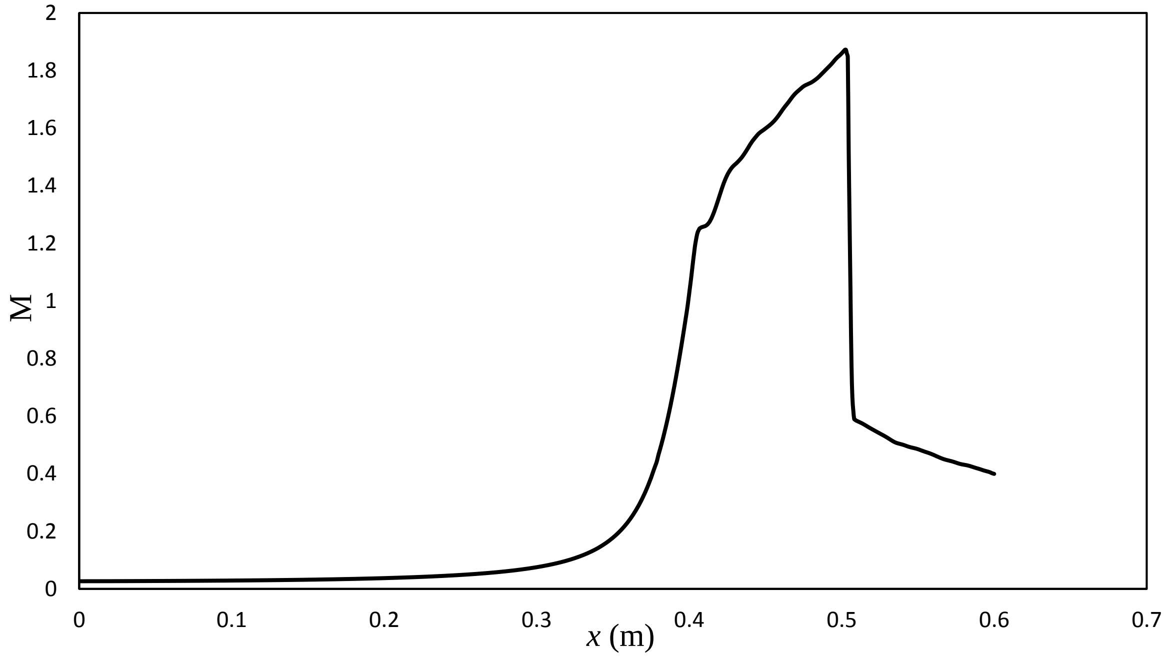

Figure 6. Variation of Mach number.

The post-shock Mach number from the plot is $M = 0.59$. The value is fairly close the one calculated using the normal shock relation $M = 0.5996$.

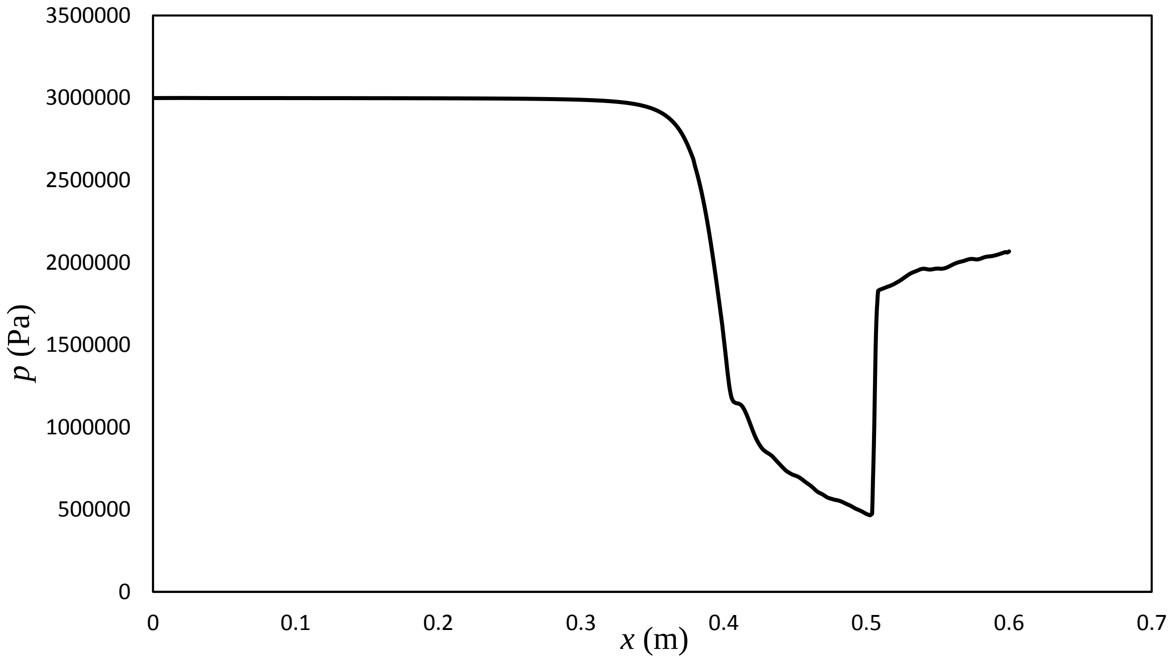

Figure 7. Variation of pressure.

The post-shock pressure is 18.3 bar, which gives $p_2/p_{o1} = 0.61$. The same calculated using the normal shock relation is $p_2/p_{o1} = 0.609$.

5. Conclusion

The flow of air through a pressure driven nozzle connected to a reservoir was simulated using OpenFOAM solver rhoCentralFoam. The simulation produced expected result. The shock location matched with the analytical solution. The Mach number and pressure changes across the shock also matched with the ones calculated using normal shock relations. The simulation also showed the flow stagnation in the reservoir.

References

1. Anderson, J.D., Modern Compressible Flow, McGraw Hill Inc., New York, 1984.

2. Greenshields, C. J., Weller, H. G., Gasparini, L. and Reese, J. M. (2010), Implementation of semi-discrete, non-staggered central schemes in a collocated, polyhedral, finite volume framework, for high-speed viscous flows. Int. J. Numer. Meth. Fluids, 63: 1-21. doi:10.1002/fld.2069

3. Zucker R, Biblarz O. Fundamentals of Gas Dynamics, Second Edition. New York: John Wiley & Sons; 2002.