Report on

Rayleigh-Bénard Convection

Case Study Project

Under the guidance of
Prof. Shivasubramanian Gopalakrishnan

Submitted by
Ashley Melvin

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1. Introduction

When a horizontal layer of fluid is heated from below, the fluid develops regular patterns called Bénard cells. This type of natural convection is called Rayleigh-Bénard convection. Because of the ease of analytical and experimental analysis, it is one of the most studied convection phenomena.

Around 1900, Bénard made some quantitative experiments on thermal convection. He melted about 1mm deep layer of wax in a metal dish by heating the base. There was no motion of liquid wax when it was heated just enough to melt all the fax. But when the temperature was raised above a certain critical temperature, Bénard observed hexagonal patterns develop on the surface of the wax and deduced the presence of convection cells below. Rayleigh modelled the problem in 1916 and treated it by the use of the theory of hydrodynamic stability.

The idea behind the Rayleigh-Bénard instability is to take a uniform homogeneous fluid sandwiched between two plates, and to heat the bottom plate so that a density gradient emerges with a cooler, denser layer lying on top of a hotter, less dense layer, thereby inducing an unstable stratification. Beyond a threshold values, this configuration becomes unstable triggering a convective motion that counteracts the unstable stratification.

2. Governing Equations

The Navier-Stokes equations of incompressible flow in an arbitrary domain is

$$\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\mu\nabla^2 u_i+\rho g_i,$$

where all symbols have their usual meaning and the gravity vector is $(g_1, g_2 . g_3) = (0, -g, 0)$ such that gravity points in the negative y-axis. The Navier-Stokes equation is supplemented with the incompressibility condition

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}(\rho u_i)=0.$$

To close the Navier-Stokes equation, further conditions are required. It is necessary to prescribe the behaviour of the density function. In the present application, i.e. the behaviour of fluid in presence of temperature gradient, it would be sensible to focus on a relationship between temperature (T) and density. The simplest model is a linear relation:

$$\rho=\rho_0+\delta\rho, \qquad \delta\rho=-\rho_0\alpha(T-T_0),$$

where ρ_0 is the reference density, $\delta\rho$ is a fluctuation which depends linearly on temperature and T_0 is the reference temperature. The quantity $\alpha > 0$ is the coefficient of volume expansion. The evolution of temperature field $T(\boldsymbol{x}, t)$ should also be specified. This can be accurately modelled by an advection-diffusion equation

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \kappa \nabla^2 T,$$

where $\kappa > 0$ is the thermal diffusivity. Assembling all equations into a single mathematical model:

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i + \rho g_i, \tag{1a}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0, \tag{1b}$$

$$\rho = \rho_0 + \delta\rho, \qquad \delta\rho = -\rho_0 \alpha (T - T_0), \tag{1c}$$

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \kappa \nabla^2 T. \tag{1d}$$

Boussinesq pointed out that there are many situations of practical occurrence in which the basic equations can be simplified considerably. These situations occur when variability of density and various coefficients is due to moderate variation in temperature. For most fluids that are considered for Rayleigh- Bénard convection, the coefficient of volume expansion (α) is small that variation in density variation need to be considered only in the buoyancy (gravity) term. This is called the Boussinesq approximation. With this approximation, equations (1a)-(1b) simplify to

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i + \left(1 + \frac{\delta\rho}{\rho_0} \right) g_i, \qquad \nu = \mu / \rho_0, \tag{2a}$$

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{2b}$$

while equations (1c) and (1d) remain unchanged.

3. Implementation in OpenFOAM

3.1. Problem Statement

The problem considers the Rayleigh-Bénard convection for two fluids. The first case is a gas whose properties (viscosity, coefficient of volume expansion, Prandtl number) resemble that of air at 300 K. In the second case, Rayleigh-Bénard convection in water at 300 K is analysed.

The temperature difference and spacing between the hot and cold plates are the same for both cases, 1 K and 1 m. In fig. 1, this would be $\theta_0 = 300$ K, $\theta_1 = 301$ K and $d = 1$ m.

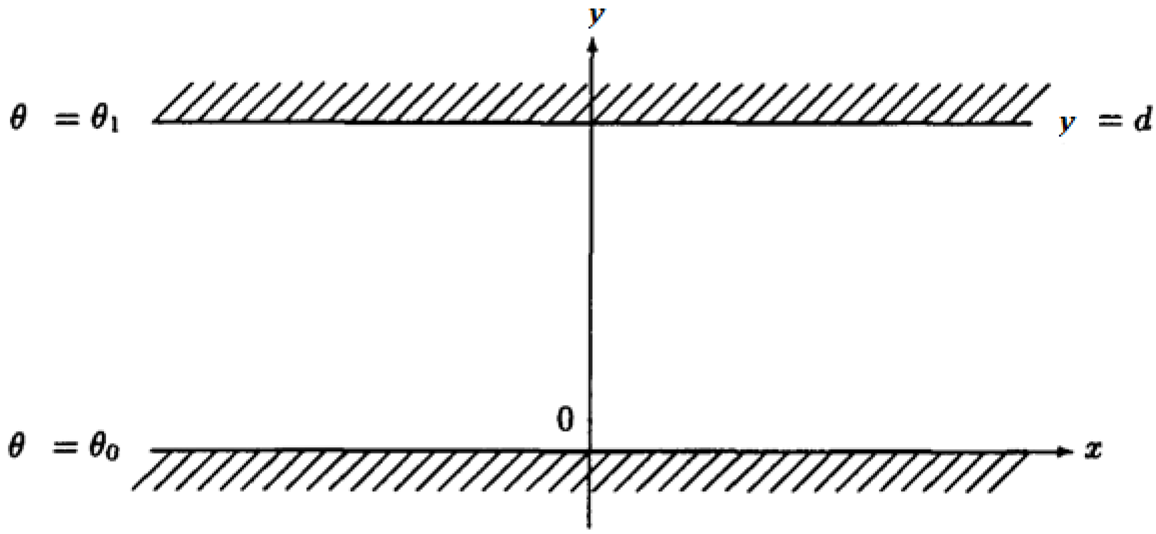

Figure 1. The configuration of Rayleigh-Bénard convection [1].

The reference temperature for the problem is taken as the average of the temperatures of two plates, i.e. 300.5 K.

3.2. Geometry & Meshing

A block of 9 x 1 x 2 m is considered. The mesh discretization used was simplegrading with 900 x 100 x 1 cells. The simulation is 2D in xy-plane. The geometry is shown in fig. 2.

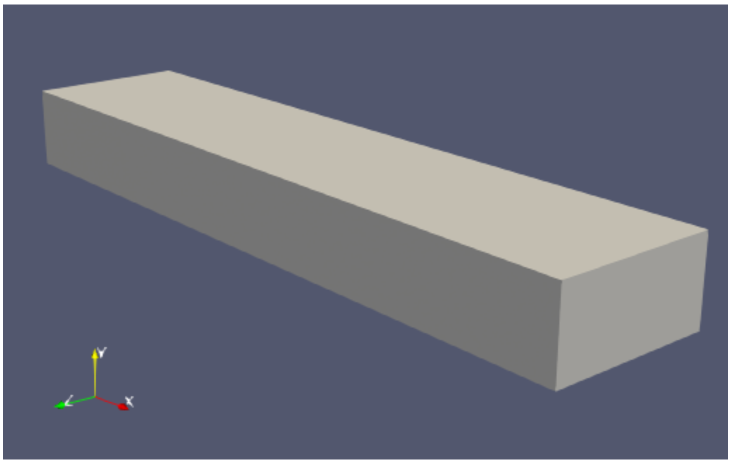

Figure 2. The geometry used for Rayleigh-Bénard convection.

3.3. Initial & Boundary Conditions

The values of various properties used in the simulation is as follows.

a) Temperature

The bottom plate is fixed at 301 K and the top plate at 300 K. The side walls are imposed with zero gradient condition.

b) Pressure

The initial internal pressure field is set to zero throughout the fluid. The values as marched in time is updated by the solver.

c) Velocity

The initial internal velocity field is set to zero throughout the fluid. The side walls as well as the top and bottom plates are imposed with no-slip condition.

The initial temperature field is shown in fig. 3.

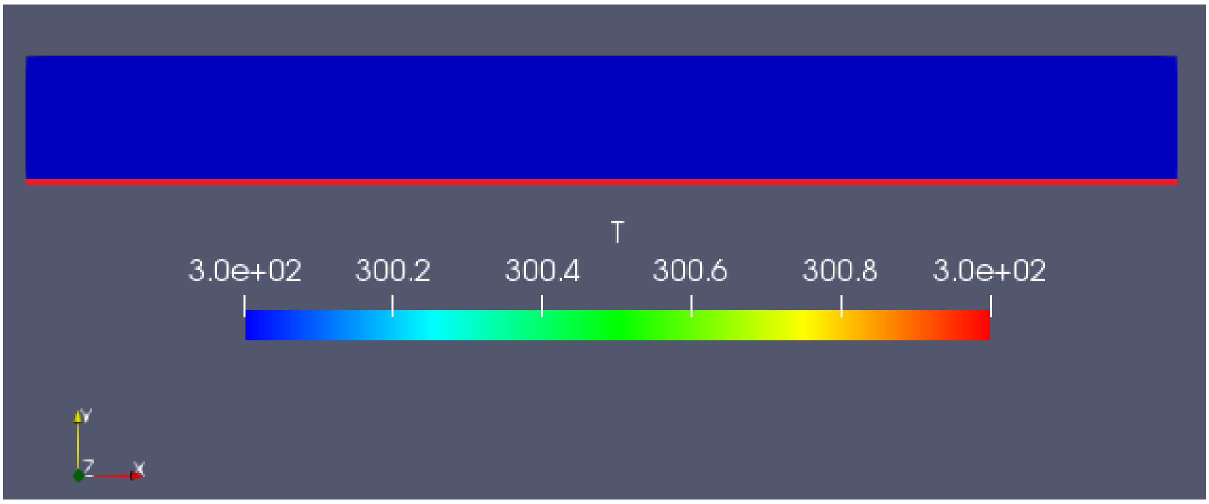

Figure 3. Initial temperature distribution.

3.4. Solver

The Rayleigh-Bénard convection governing equations, (2a)-(2b) and (1c)-1(d), are solved using buoyantBoussinesqPimpleFoam. This is one of the many solvers for solving heat transfer problems in OpenFOAM. In this particular solver, the heat transfer problems are solved in presence of gravity body force. The mass and momentum equations are as derived in section 2. This solver also uses the Boussinesq approximation to simplify the mass and momentum equations. The k-Epsilon model is used for turbulence modelling in the simulation.

4. Results

The simulations are run on OpenFOAM 5.0 and the post processing is done using ParaView.

4.1. Case 1: Air

The temperature and velocity fields in the xy-plane are shown in fig. (4) and (5).

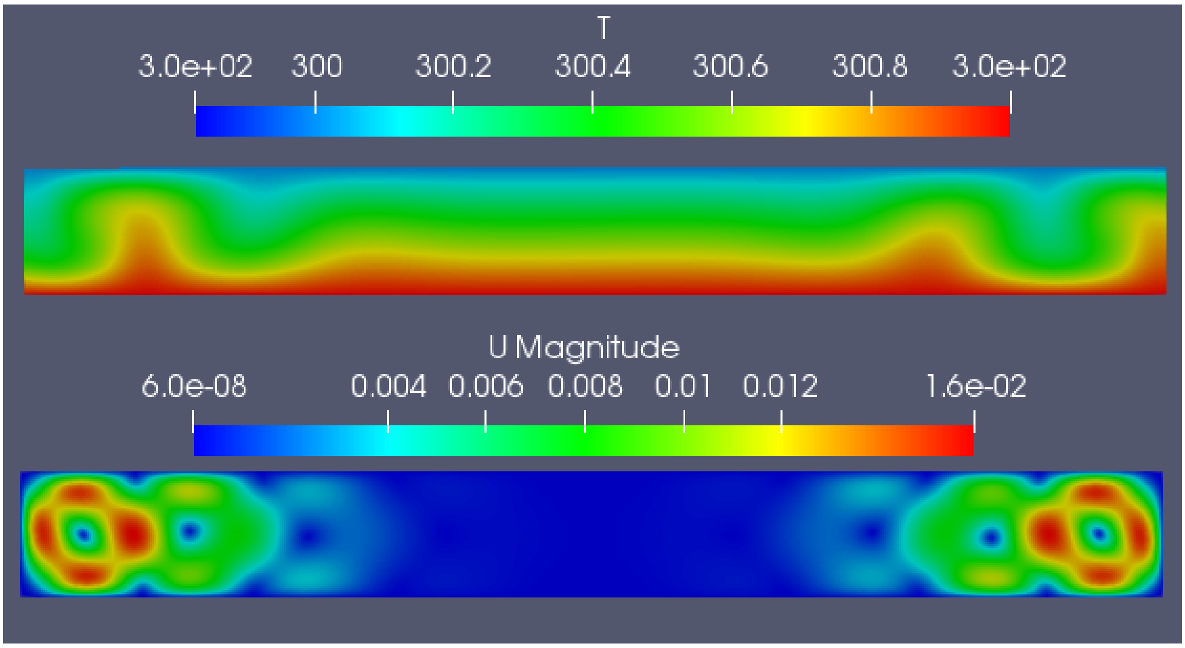

Figure 4. Temperature and velocity fields at t = 800 seconds.

The velocity field in fig. 4 clearly captures the onset of Bénard cell formation in the fluid.

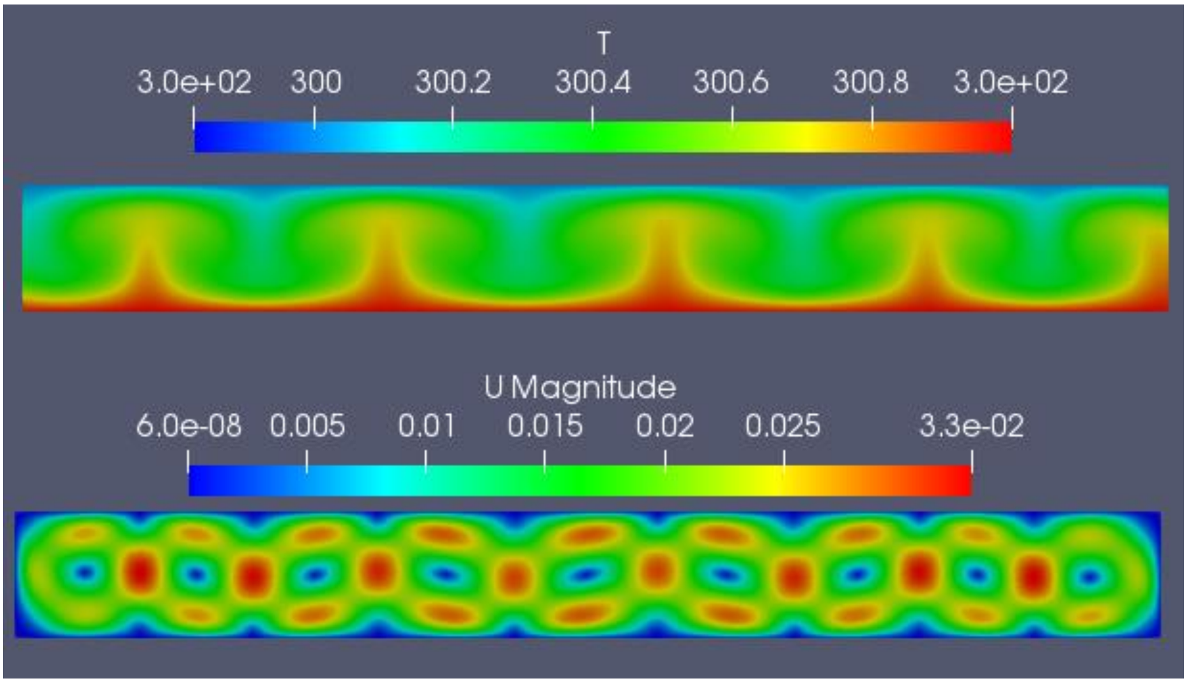

Figure 5. Temperature and velocity fields at t = 3000 seconds.

The streamlines in the xy-plane are shown in fig. (6).

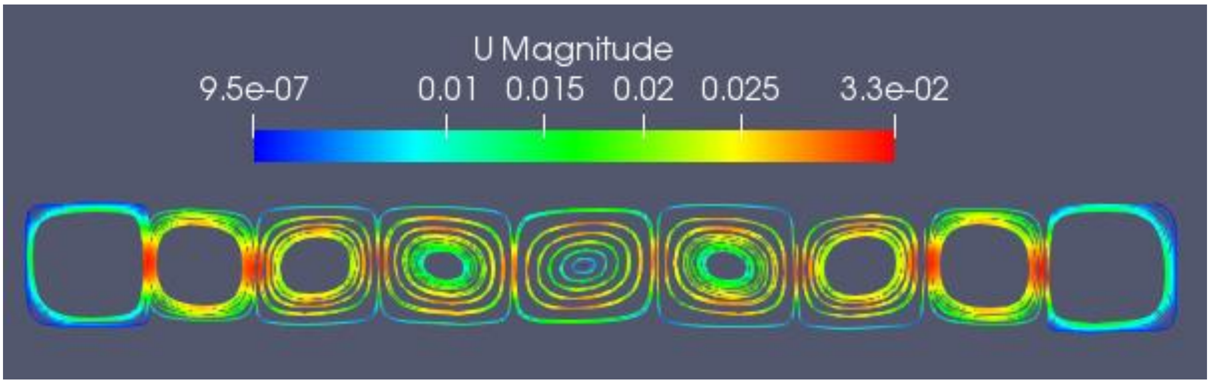

Figure 6. Streamlines at t = 3000 seconds.

The streamlines clearly indicate the presence of 9 Bénard cells in the xy-plane.

4.2. Case 2: Water

The temperature and velocity fields in the xy-plane are shown in fig. (7).

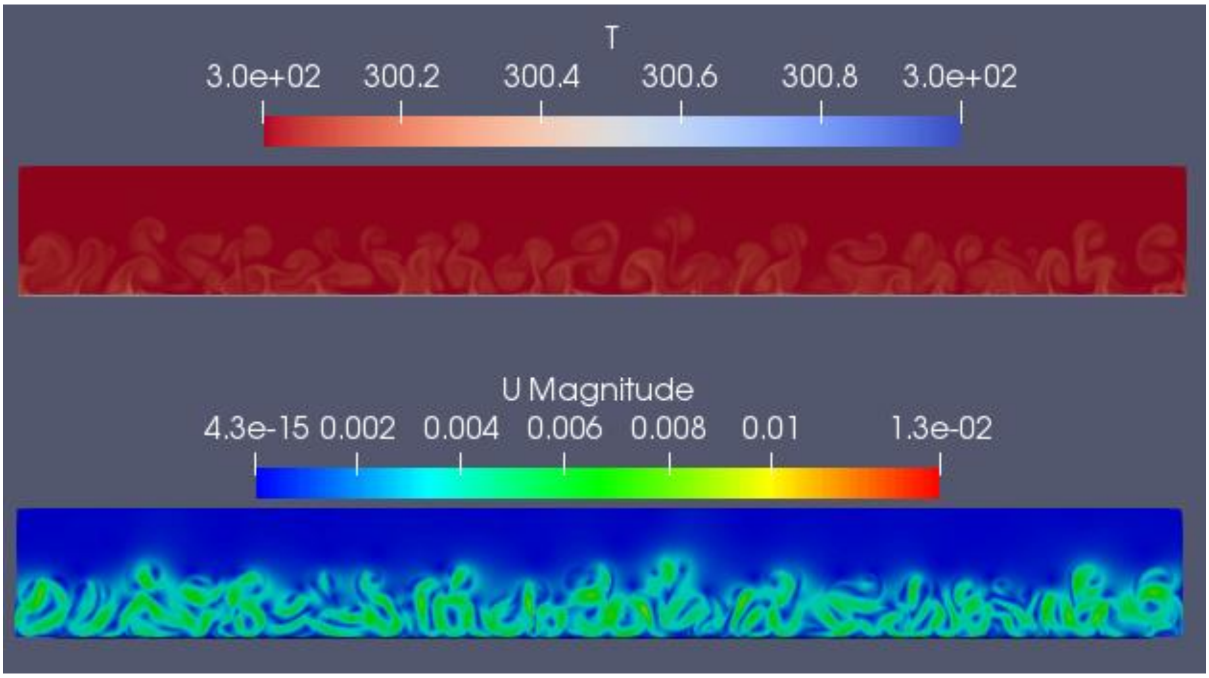

Figure 7. Temperature and velocity fields at t = 600 seconds.

The temperature and velocity fields in fig. 7 captures the onset of convection in water.

The streamlines in the xy-plane are shown in fig. 8. The streamlines clearly indicate the presence of 9 Bénard cells in the xy-plane. The cells are distorted compared to ones observed in the previous case (air).

The temperature and velocity fields at the same time are shown in fig. (9).

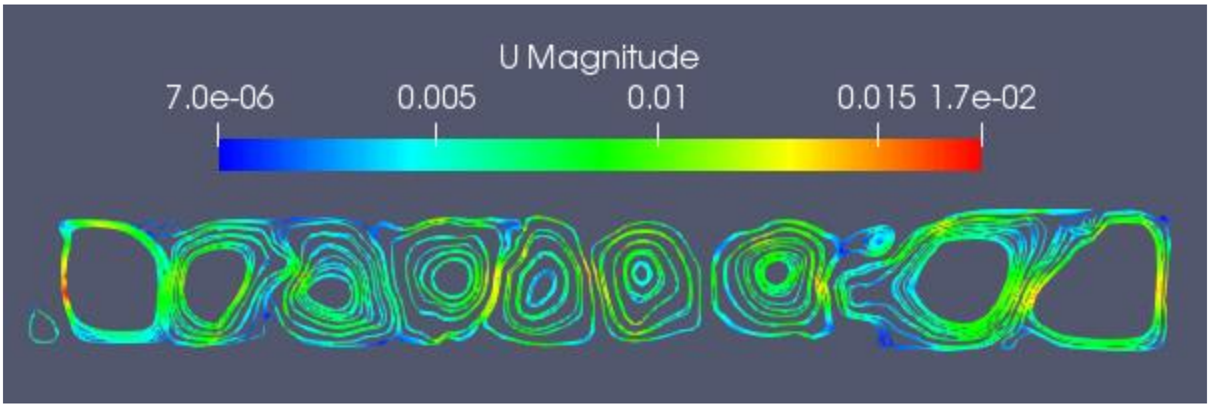

Figure 8. Streamlines at t = 1800 seconds.

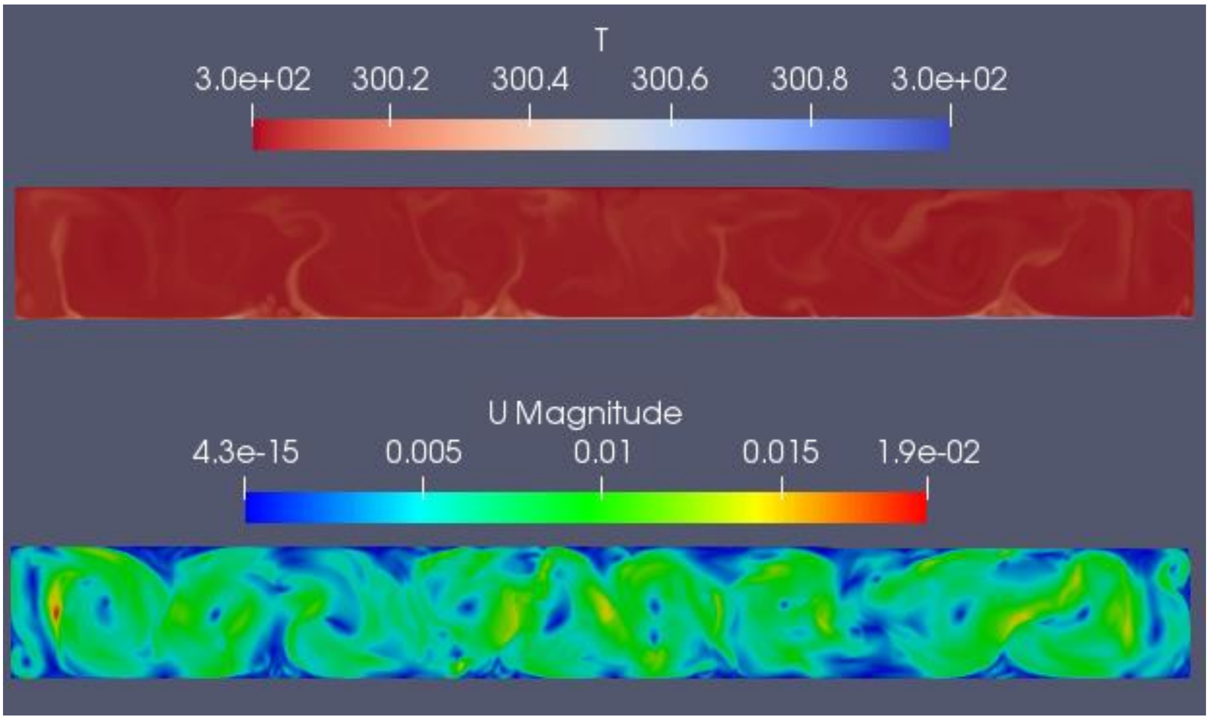

Figure 9. Temperature and velocity fields at t = 1800 seconds.

5. Conclusion

The Rayleigh-Bénard instability develops when the Rayleigh number of the fluid exceeds a certain critical value. In both the cases considered (rigid-rigid), that value predicted by linear stability analysis [2] is 1708. Therefore, while considering the temperature differences in both the cases, care was taken so that the Rayleigh number is greater than the critical value. Since the Boussinesq approximation is valid only for small temperature difference, the temperature difference was chosen to be 1 K which also gives a Rayleigh number greater than 1708.

The formation of Bénard cells are well captured in both cases. The number of cells is proportional to the aspect ratio used. An aspect ratio of 9 yielded 9 cells in the plane. Also the temperature distribution compares well with the experimental results.

References

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2. Subrahmanyan Chandrasekhar (1982). Hydrodynamic and Hydromagnetic Stability (Dover). ISBN 0-486-64071-X

3. Lennon Ó Náraigh (2017). Rayleigh–Bénard Convection-Linear Theory. School of Mathematics and Statistics, University College Dublin, Ireland.