Effect of Convexity on onset of Rayleigh-Bénard Convection in Convex and Concave Cylinders

Govind Maurya

Department of Energy Science and Engineering, IIT Bombay

Abstract

The objective of this study is to conduct a simulation of the onset of Rayleigh-Bénard convection in a cylindrical cavity of either concave or convex shape. A piece of the cavity, having center line as axisymmetric is simulated for that purpose. Rayleigh-Bénard Convection is a special type of natural convection having thermal gradient anti-parallel to the gravity which occurs when thermally driven buoyancy dominates over viscosity force resulting in the formation of convective rolls after the onset. These rolls possess a structure that is ideal for thermally activated polymerized chemical chain reactions through temperature cycling, [1]. Current work investigates the stability dependence of RBC on convexity of cylindrical cavity.

1. Introduction

Rayleigh-Bénard convection is a phenomenon in fluid dynamics that occurs when a fluid is heated from below, resulting in the formation of convective cells. This causes the fluid to start circulating, with hot fluid rising and cool fluid sinking, creating convection cells. This phenomenon has been studied extensively in various geometries. In a square cavity, Rayleigh-Bénard convection can be particularly interesting to observe due to the confined geometry and the resulting complex flow patterns. The onset of Rayleigh-Bénard convection in a square cavity is a critical point beyond which the convective flow dominates over conduction and it has been the subject of much research in fluid mechanics. Understanding the onset of Rayleigh-Bénard convection in a square cavity is essential for a wide range of applications, such as, the inherent structure of convective rolls suites for performing thermally activated polymerised chemical chain reactions by temperature cycling [1]. In this work, OpenFoam simulation of the onset of Rayleigh-Bénard convection in a square cavity and convex and concave shaped cylindrical cavity has been done. Benchmark of OpenFoam simulations have been done works on Rayleigh-Bénard Convection in square cavities [2,3]. Current work investigates the stability dependence of RBC on convexity of cylindrical cavity.

2. Problem Statement

The square cavity is solved in OpenFoam with bottom heated, top cooled and adiabatic side walls. The results are benchmarked with Ouertatani et al. [2] and Venturi et. al. [3] and found in good agreement with these. After the benchmark, the concave and convex shaped cylinders have been simulated in OpenFoam. A schematic diagram of the concave cylindrical cavity has been shown which can be rotated 360 degree to form a concave cylindrical cavity. Similarly, the convex cylindrical cavity, having curved wall outside, was also simulated. The bottom of the cavity was heated and top wall was cooled. The curved walls of cylindrical cavity were taken adiabatic while the left straight wall is line of axisymmetric.

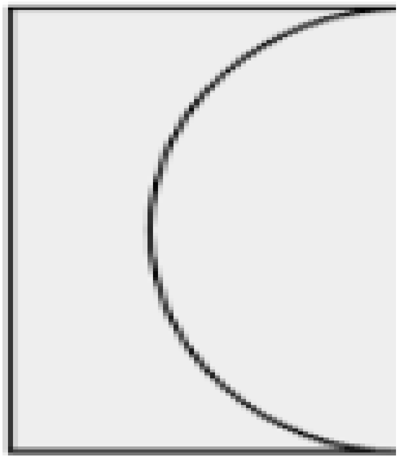

Fig-1: The schematic diagram of the portion of cavity which can be rotated 360 degree to get concave cylindrical cavity

3. Governing Equations

The governing equations solved using OpenFOAM are incompressible form of Navier-Stokes equation along with continuity and energy equation in advection-diffusion form. The Boussinesq approximation was considered which states that the density variation linear in terms of temperature can be considered only in the buoyancy (term having gravity) while density in all other terms is taken to be constant due to smaller thermal expansion coefficient in the temperature range.

4. Simulation Procedure

4.1 Geometry and Mesh

Initially the square cavity is created in OpenFoam for benchmark simulation. Later, the convex and concave part of cylindrical cavity was created and the mesh of these cavities are shown as follows for 20x20.

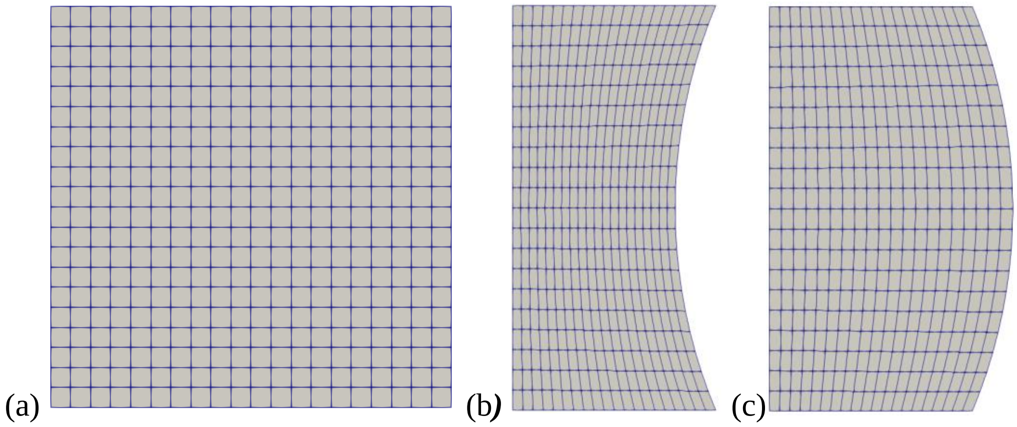

Fig-2: The mesh diagram of square cavity (a), concave (b) and convex cylindrical cavity (c) respectively.

4.2 Initial and Boundary Conditions

The initial internal velocity field is set to zero throughout the fluid as there is no initial flow in the cavity. The side walls as well as the top and bottom plates are imposed with no-slip condition.

The thermal boundary condition for top wall was T = 300K, and for bottom wall T = 320K and adiabatic boundary conditions of zero gradient on side walls were same for all 3 cavities as given below except in case of convex and concave cavities, the axisymmetry boundary condition was applied on vertical straight wall due to cylindrical symmetry. The initial internal pressure field was set to zero throughout the fluid. The values as marched in time is updated by the solver.

Top Wall: U = 0, V = 0, T = 300K

Bottom Wall: U = 0, V = 0, T = 320K

Curved or Side Wall: Adiabatic Condition

Straight Vertical Wall in Cylindrical Cavities: Axisymmetric

4.3 Solver

The buoyantPimpleFoam solver used in the openFoam 7.0 for simulation works done. In this particular solver, the heat transfer problems are solved in presence of gravity force. This solver uses the Boussinesq approximation to simplify the mass and momentum equations. The flow was considered in the laminar region and thus solver was modified for laminar flow only.

5. Results and Discussions

5.1 Verification with Ouertatani

The Rayleigh number can be changed by changing either temperature of bottom wall or the size of cavity. For larger bottom temperature, there was some simulation error, that is why the Rayleigh number is varied by changing cavity size. Rest of the parameters in calculation of Ra are taken for the air at mean temperature 310 K. The simulation has been done in square cavity for Rayleigh number 10000 and 100000 to match the results with Ouertatani et al. results. The isotherms of Ouertatani et al. and from current simulation has been shown below which matches very well.

The non-dimensionalised form of maximum horizontal and vertical velocities have been compared also with Ouertatani et. al. and it is found that these values match very well with their results as shown in table below. Since current work only focus in laminar region, we restricted simulation till Ra= 100000 only.

Table-1: The comparisons of the maximum value of non-dimensionalised horizontal and verical velocities in square cavity for Ra = 10000 and 100000 respectively from Ouertatani et. al. [2] and current work

	Ra	***Umax***	***Vmax***
Ouertatani	10000	0.252	0.264
	100000	0.344	0.376
Current Work	10000	0.254	0.275
	100000	0.312	0.352

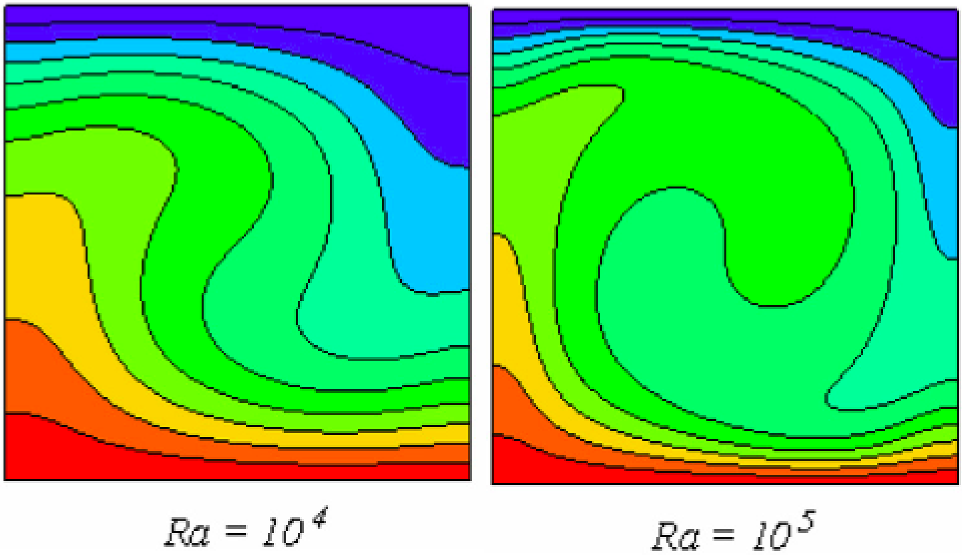

Fig-3: The isotherms in square cavity for Ra = 10000 and 100000 respectively from Ouertatani et. al. [2]

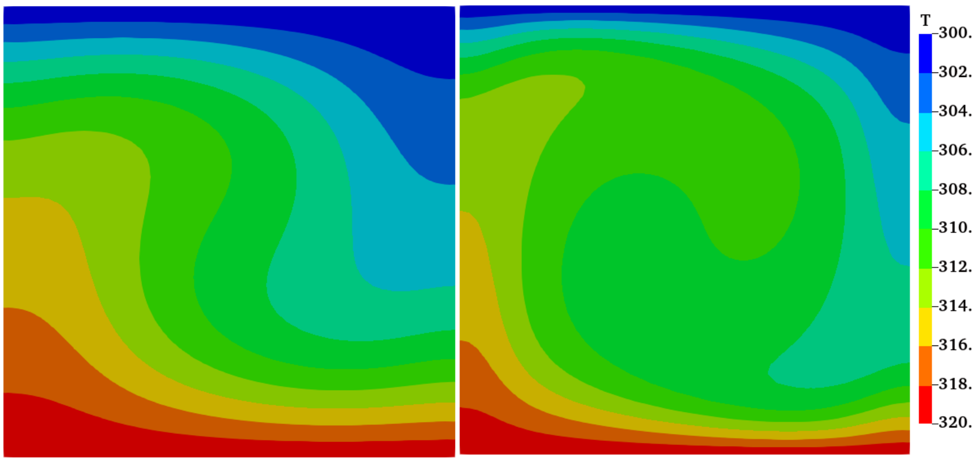

Fig-3: The isotherms in square cavity for Ra = 10000 and 100000 respectively from current work

5.2 Verification with Venturi

Venturi et al. have calculated critical Rayleigh number in square cavity using Galerkin spectral method and presented the Rac =2585.02 [3]. As it is simulation work, the result can't match exactly with analytical works, but within 5 percent error, the onset of Rayleigh-Bénard Convection between Ra=2600-2700 is observed. The onset is indicated by both isotherm profile and sudden jump in the maximum of Velocity magnitude. The isotherms are shown as below.

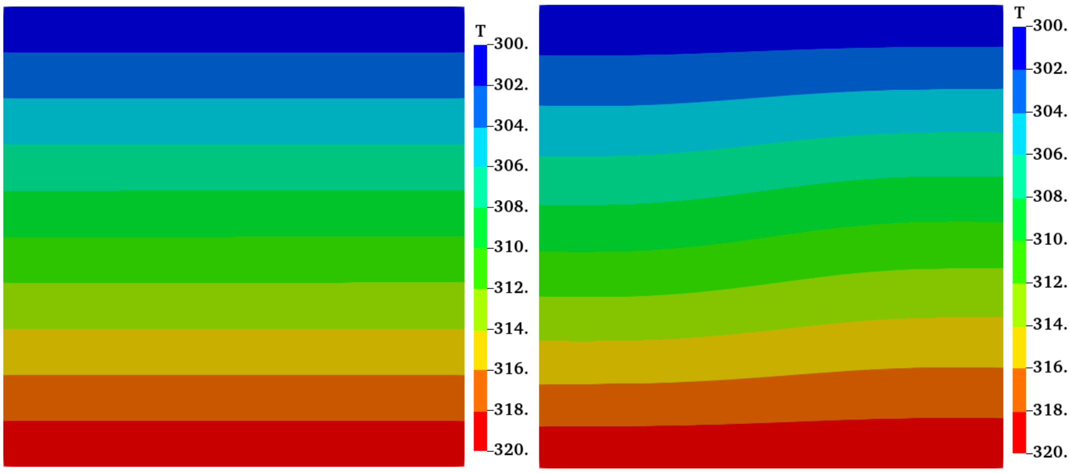

Fig-4: The isotherm in square cavity showing onset of Rayleigh-Bénard Convection for Ra = 2600 and 2700 respectively.

The velocity magnitude contours indicate that the cavity is having low flow surrounded by high flow velocity. The maximum value of the velocity magnitude increased more than 80 times by changing 100 Rayleigh number, which is clearly indicating the transition from almost zero flow to flow i.e. onset of convection.

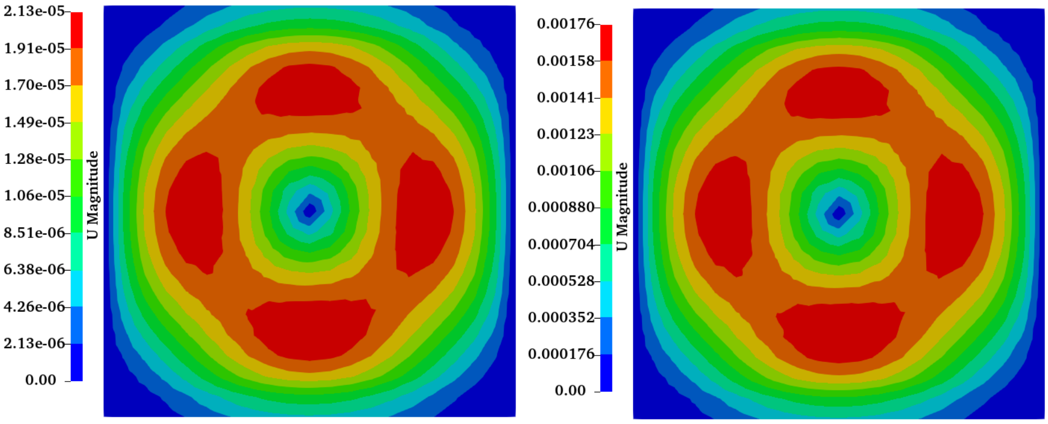

Fig-5: *The velocity magnitude contours in square cavity for Ra = 2600 and 2700 respectively.*

Also, the isotherm matched with Venturi et al for Ra=15000. The author has predicted both kind of mirror image anti-symmetric solutions. which are shown as:

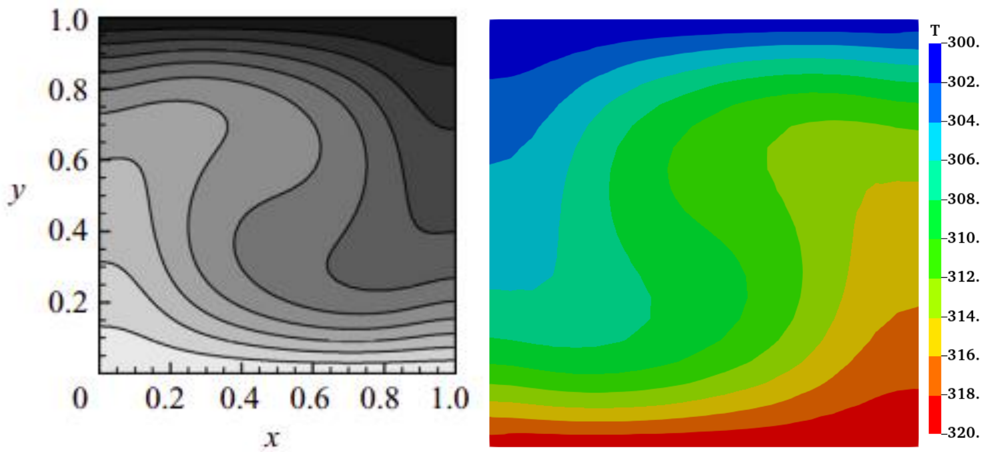

Fig-6: The isotherms in square cavity for Ra = 15000 from Venturi et. al. [3] and from current work respectively.

5.3 Results of Convex Cylindrical Cavity

The onset of Rayleigh-Bénard Convection was found for Ra=4500-4600. The isotherms and velocity magnitude contours have been presented. The onset is indicated by both isotherm profile and sudden jump in the maximum of Velocity magnitude.

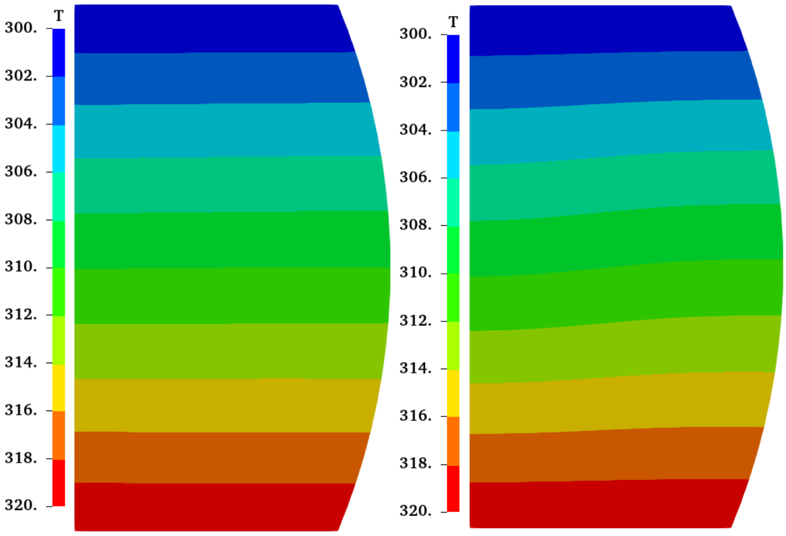

Fig-7: The isotherms in convex cylindrical cavity for Ra = 4500 and 4600 respectively.

The velocity magnitude contours indicate that the cavity is having center dominated flow surrounded by less flow velocity. The maximum value of the velocity magnitude increased almost 10 times by changing just 100 Rayleigh number, which is clearly indicating the transition and stability breakdown of Rayleigh-Bénard Convection.

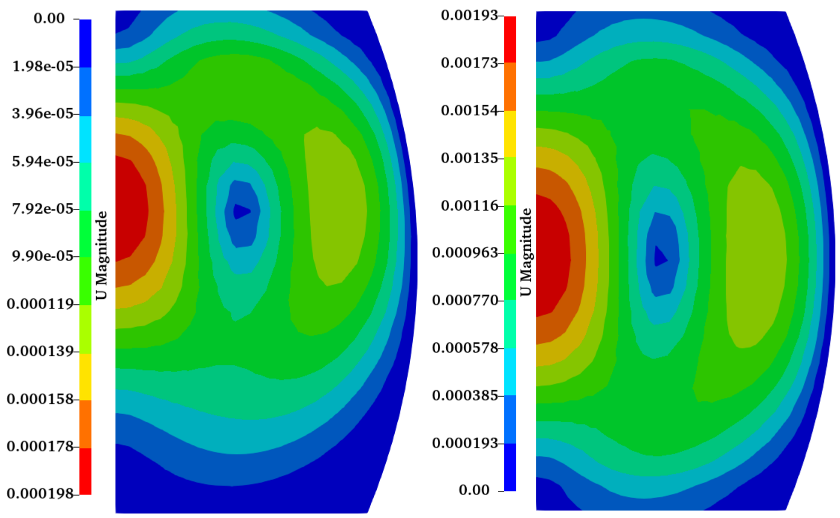

Fig-8: The velocity magnitude contours in convex cylindrical cavity for Ra = 4500 and 4600 respectively.

5.4 Results of Concave Cylindrical Cavity

The onset of Rayleigh-Bénard Convection was found for Ra=12600 - 12700. The isotherms and velocity magnitude contours have been presented. The onset is indicated by both isotherm profile and sudden jump in the maximum of Velocity magnitude.

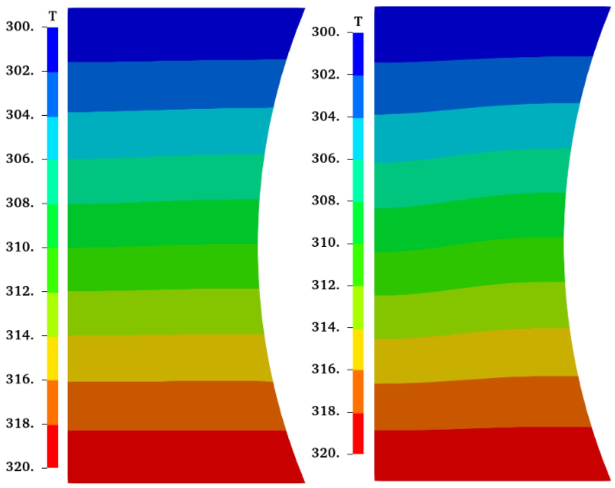

Fig-9: The isotherms in concave cylindrical cavity for Ra = 12600 and 12700 respectively.

The velocity magnitude contours indicate that the cavity is having center dominated flow surrounded by less flow velocity. The maximum value of the velocity magnitude increased almost 4 times by changing just 100 Rayleigh number, which is clearly indicating the transition and stability breakdown of Rayleigh-Bénard Convection.

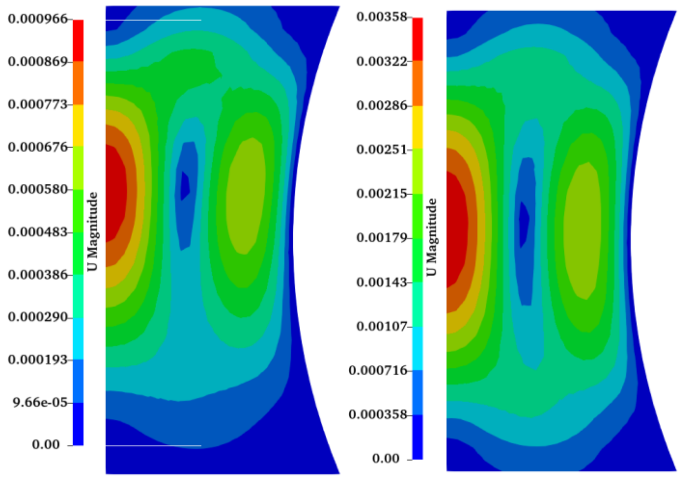

Fig-10: The velocity magnitude contours in concave cylindrical cavity for Ra = 12600 and 12700 respectively.

From these simulations for square cavity, convex cylindrical cavity and concave cylindrical cavity, it is observed that the critical Rayleigh number is increased when curved side wall is attached. Concave cylindrical cavities are having stablised flow for higher Rayleigh number than convex cylindrical cavities. Although the simulation was done for same curvature of concave and convex. So, it is deduced that the concave shapes of same aspect ratio and curvature results in more stablised flow for larger Rayleigh number than convex shapes.

References

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3. DANIELE VENTURI, XIAOLIANG WAN, GEORGE EM KARNIADAKIS; "Stochastic bifurcation analysis of Rayleigh–Bénard convection". Journal of Fluid Mechanics, Volume 650, 10 May 2010, pp. 391 - 413. DOI: https://doi.org/10.1017/S002211200999368