

# **Transient flow analysis of Constant and Time Varying inlet flow conditions through forward and reverse flow through Tesla Valve**

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## **Abstract**

This Case Study aims to highlight the differences in nature of flow between constant and time varying inlet flow rates and attempts to discern the reasons behind those differences. First the flow analysis for constant flow velocities of 2 m/sec and 6 m/sec is performed to confirm diodic nature of fluid flow through Tesla Valve. In the next step of the Case study, Inlet Flow Rate Ramp function and oscillating sinusoidal function are introduced as time dependent inlet boundary conditions for forward and reverse flow cases. Pressure, velocity and passive scalar profiles have been assessed for all the cases stated above.

## **1. Introduction**

A Tesla Valve, also called a Valvular Conduit, is a fixed geometry passive check valve. It allows the fluid to flow preferentially in one direction with no moving parts.

Since the pressure drop in one direction is more than the pressure drop in the other direction, the efficiency of the Tesla Valve is measured in terms of the ratio of the pressure difference developed in the forward and backward direction as:

$$Di = \left( \frac{\Delta P_r}{\Delta P_f} \right)_Q \text{ for the same flow rate } Q \quad \text{Eq. 1}$$

As we shall see for steady state flow state, the Diodicity is maintained between Forward and Reverse flow for all velocities considered.

Due to the equivalence between a diode and a Tesla Valve, this paper seeks to further assess Tesla Valve with Time Varying Inlet flow conditions, in order to find the extent to which the equivalence holds true.

We will observe that for each of the Cases, for both steady and transient flow, the pressure difference between the inlet and outlet of the Tesla Valve takes the following form:

$$(P_{outlet} - P_{inlet}) = k_1(U^2) + k_2\left(\frac{\partial U}{\partial t}\right) + k_3(U) \quad Eq. 2$$

The form of this equation is to consider the effect of advective terms, time gradient of velocity and the viscous forces on the pressure difference. This equation is discussed at length in Section 5 of this report.

## 2. Problem Statement

In this study, fluid flow in a T45-R Tesla Valve model with nine diodic separation branches in mm scale was studied for forward and reverse flow under steady and transient inlet flow conditions.

The following inlet flow conditions were considered for Forward and Reverse flow through Tesla Valve for this study:

- 1) Constant Inlet Flow Velocity of 2 m/s and 6 m/s from 0 to 5 seconds – Confirming the diodicity of the Tesla Valve.
- 2) Pulsating Inlet Flow Velocity of  $[5+1\sin(40\pi t)]$  m/sec from 0 to 6 seconds – To study the effect of pulsating flow on Tesla Valve.
- 3) Ramped Inlet Flow Velocity, ramped as follows:
  - a) From 0 to 2.5 secs, the inlet velocity is held constant at 2 m/s (Simulation begins)
  - b) From 2.5 secs to 5 secs, the inlet velocity was ramped from 2 m/s to 4 m/s
  - c) From 5 secs to 7.5 secs, the inlet velocity was kept constant 4 m/s
  - d) From 7.5 secs to 8.75 secs, the inlet velocity was ramped from 4 m/s to 6 m/s
  - e) From 8.75 secs to 12.5 secs, the fluid flow rate was kept constant at 6 m/s. (End of Simulation).

Reynold's number was  $4.35 \times 10^5$ . Thus, since the fluid flow was in turbulent regime and transient flow characteristics are being studied, pisoFoam solver with k-epsilon turbulent flow model was used.

### 3. Governing Equations

The following incompressible Navier Stokes equations were used:

- 1) Continuity equation for mass flow rate conservation:

$$\nabla \cdot U = 0$$

- 2) Momentum equation:

$$\left(\frac{\partial U}{\partial t}\right) + \nabla \cdot (UU) = -\left(\frac{1}{\rho}\right)\nabla p + \mu \nabla^2 U + g$$

$U$  is the Velocity Vector,  $\rho$  is the density of fluid,  $\mu$  is the Dynamic Viscosity,  $P$  is the Pressure and  $g$  is the gravitational acceleration constant.

In this Case Study, a two dimensional simulation is being carried out (in X and Y direction). The gravitational acceleration constant is not used in this case.

### 4. Simulation Procedure

#### 4.1 Geometry and Mesh

The geometry and meshing was done in Salome Meca as shown in Figure 1.

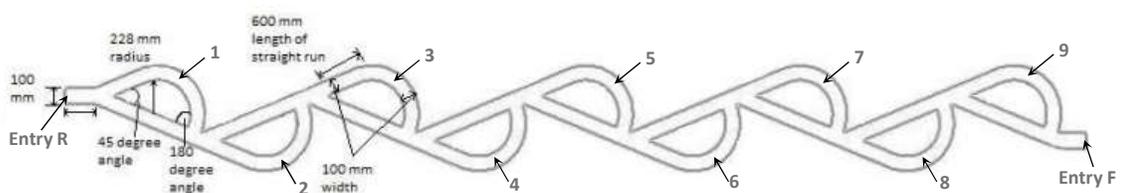


Fig. 1: Dimensions of the Tesla Valve Geometry. The nine diode separations have the same dimensions and vary only in orientation to one another, and are labelled from 1 to 9 as shown. Entry R and Entry F refer to each end of the geometry through which entry occurs for reverse and forward flow respectively.

Hexahedral 3D meshing with 2-D Quadrangle mapping was done (Fig. 2). The mesh had a total of 33168 nodes, which meant each node had a length of 10 mm. Considering a channel

size of 100 mm, this gave a total of 10 nodes across the channel width. A single node width of 10 mm was considered in the Z direction as this is a 2-D simulation.

Since Salome Meca does not state units for the dimensions, and OpenFoam assumes standard units of metres, TransformPoints scale was used to scale the mesh from m units to mm units.

The ideal timestep for the case study was found to range from  $0.5 \times 10^{-4}$  seconds to  $1 \times 10^{-4}$  seconds to ensure stability of pisoFoam simulation – keeping Courant number less than 1.

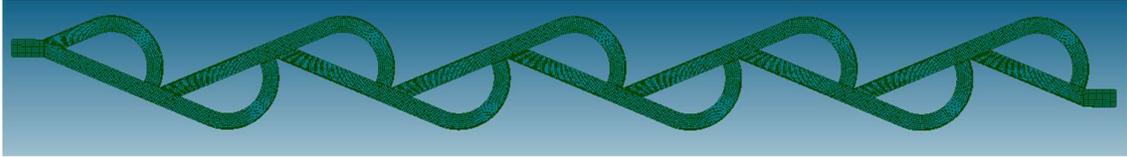


Fig. 2: Hexahedral meshing in Salome Meca.

## 4.2 Initial and Boundary Conditions

The following Boundary Conditions were implemented for forward and reverse flow:

Case 1: Boundary Conditions for constant flow velocities of 2 m/s and 6 m/s for forward and reverse cases in the X-direction are as tabulated in Table 4.1. The constant flow velocity cases have been simulated for 0 to 5 sec. A passive scalar of magnitude 1 has been considered in the simulation to track the fluid flow from its point of advection in the Tesla Valve.

Table 4.1: Boundary Conditions for constant flow velocities of 2 m/s and 6 m/s for forward and reverse cases

Boundary Condition	Reverse Flow			Forward Flow		
	U (m/sec)	S (Scalar)	P ( $\text{kg}^2/\text{sec}^2$ )	U (m/sec)	S (Scalar)	P ( $\text{kg}^2/\text{sec}^2$ )
Inlet	fixedValue uniform	fixedValue uniform	zeroGradient	zeroGradient	zeroGradient	fixedValue uniform ( $0 \text{ kg}^2/\text{sec}^2$ )
Outlet	zeroGradient	zeroGradient	fixedValue uniform ( $0 \text{ kg}^2/\text{sec}^2$ )	fixedValue uniform	fixedValue uniform	zeroGradient
Walls	noSlip	zeroGradient	zeroGradient	noSlip	zeroGradient	zeroGradient
Front And Back	Empty	Empty	Empty	Empty	Empty	Empty

Case 2: Boundary Conditions for pulsating inlet flow velocity  $[5+1\sin(40\pi t)]$  m/sec in the X direction for forward and reverse flow are as tabulated in Table 4.2. The pulsating inlet flow case has been simulated from 0 sec to 6 sec. A passive scalar has been introduced at its starting point of advection (Entry F for forward flow, and Entry R for Reverse flow) from 0 to 3 seconds to track the fluid from start of simulation till the scalar reaches the other end of the Tesla Valve. After 3 seconds, the passive scalar has been turned off, to track how long it takes for the passive scalar to dissipate completely.

Table 4.2: Boundary Conditions for pulsating inlet flow velocity  $[5+1\sin(40\pi t)]$  m/sec for forward and reverse cases.

Boundary Condition	Reverse Flow			Forward Flow		
	U (m/sec)	S (Scalar)	P ( $\text{kg}^2/\text{sec}^2$ )	U (m/sec)	S (Scalar)	P ( $\text{kg}^2/\text{sec}^2$ )
Inlet	Uniform-FixedValue (sinusoidal boundary condition)	Uniform-Fixed-Value (0 to 3 secs)	zeroGradient	zeroGradient	zeroGradient	Fixed-Value uniform ( $0 \text{ kg}^2/\text{sec}^2$ )
Outlet	zeroGradient	zeroGradient	fixedValue uniform ( $0 \text{ kg}^2/\text{sec}^2$ )	Uniform-FixedValue (sinusoidal boundary condition)	UniformFixed-Value (0 to 3 secs)	Zero-Gradient
Walls	noSlip	Zero-Gradient	Zero-Gradient	noSlip	Zero-Gradient	Zero-Gradient
Front And Back	Empty	Empty	Empty	Empty	Empty	Empty

Case 3: Boundary Conditions for ramped inlet flow velocity in the X direction in both forward and reverse flow are as tabulated in Table 4.3. The inlet flow velocity has been ramped as follows:

- a) From 0 to 2.5 secs, the inlet velocity is held constant at 2 m/s (Simulation begins)
- b) From 2.5 secs to 5 secs, the inlet velocity was ramped from 2 m/s to 4 m/s
- c) From 5 secs to 7.5 secs, the inlet velocity was kept constant 4 m/s
- d) From 7.5 secs to 8.75 secs, the inlet velocity was ramped from 4 m/s to 6 m/s
- e) From 8.75 secs to 12.5 secs, the fluid flow rate was kept constant at 6 m/s. (End of Simulation).

Table 4.3: Boundary Conditions for ramped inlet flow velocity (profile discussed above) for forward and reverse cases.

Boundary Condition	Reverse Flow			Forward Flow		
	U (m/sec)	S (Scalar)	P (kg <sup>2</sup> /sec <sup>2</sup> )	U (m/sec)	S (Scalar)	P (kg <sup>2</sup> /sec <sup>2</sup> )
Inlet	Uniform-FixedValue (tabulated boundary condition – profile as discussed above)	Uniform-Fixed-Value (Five scalars as described above)	zeroGradient	zeroGradient	zeroGradient	fixedValue uniform (0 kg <sup>2</sup> /sec <sup>2</sup> )
Outlet	zeroGradient	zeroGradient	fixedValue uniform (0 kg <sup>2</sup> /sec <sup>2</sup> )	Uniform-FixedValue (tabulated boundary condition – profile as discussed above)	Uniform-Fixed-Value (Five scalars as described above)	zeroGradient
Walls	noSlip	Zero-Gradient	Zero-Gradient	noSlip	Zero-Gradient	Zero-Gradient

Boundary Condition	Reverse Flow			Forward Flow		
	U (m/sec)	S (Scalar)	P (kg <sup>2</sup> /sec <sup>2</sup> )	U (m/sec)	S (Scalar)	P (kg <sup>2</sup> /sec <sup>2</sup> )
Front And Back	Empty	Empty	Empty	Empty	Empty	Empty

A passive scalar was introduced at the commencement of each of these stages to track the nature of changing fluid flow in the Tesla Valve.

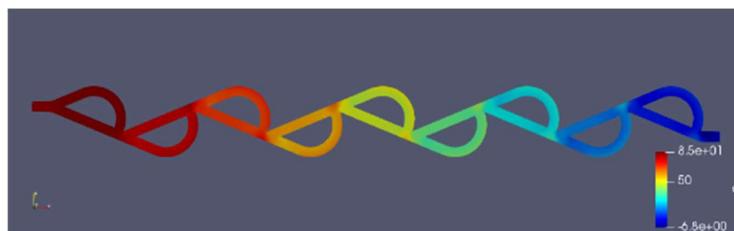
### 4.3 Solver

The solver used is pisoFoam, which is a transient solver for incompressible flow. Considering that the Reynold's number for the lowest inlet velocity of 2 m/s is  $4.35 \times 10^5$ , for all the cases, the flow is in turbulent regime. Thus, Reynolds Averaged k-Epsilon model was used for all the simulation cases.

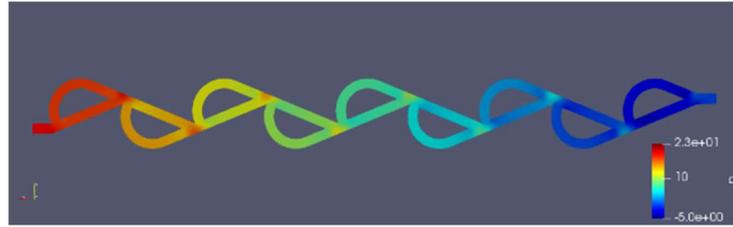
## 5. Results and Discussions

Case 1: Constant flow velocities of 2 m/s and 6 m/s for forward and reverse cases in the X-direction.

First, to confirm diodic nature of Tesla Valve, a comparison of the pressure profile in the forward and reverse direction for the 2 m/s and 6 m/s cases at the end of 5 seconds simulation are as shown in Figures 3 and 4 respectively. For this, a probe was located at [0,0.05, 0.0005] for reverse flow, and at [7.025,-0.25,0.0005] for forward flow case.



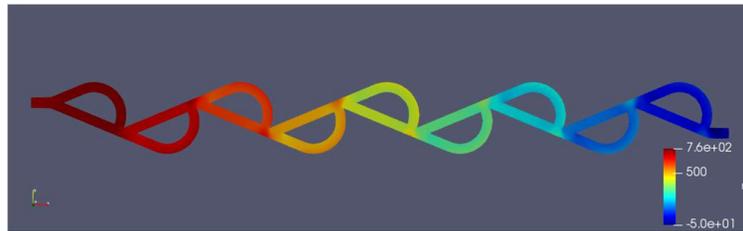
(a)



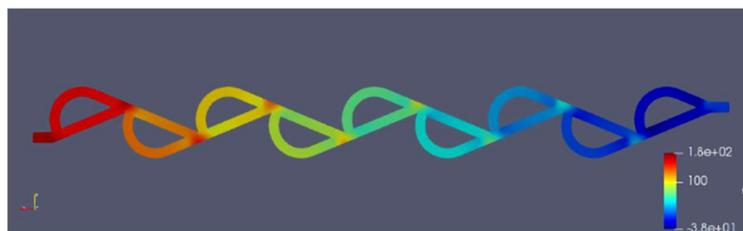
(b)

Fig 3 Pressure Profile for (a) reverse flow and (b) forward flow (2 m/s constant velocity)

In the 2 m/s case, at the end of 5 seconds, pressure difference in reverse flow case was found to be  $83.23 \text{ m}^2/\text{s}^2$  and for forward flow case was found to be  $19.11 \text{ m}^2/\text{s}^2$ . Thus, diodicity of Tesla Valve came out to be 4.3. For the 6 m/s case, at the end of 5 seconds, the pressure difference in the reverse flow case was found to be  $741.8 \text{ m}^2/\text{s}^2$  while the pressure difference in the forward flow case was found to be  $170.131 \text{ m}^2/\text{s}^2$  giving a diodicity of 4.3. The diodicity of the Tesla Valve thus turns out to be 4.3.



(a)



(b)

Fig 4 Pressure Profile for (a) reverse flow and (b) forward flow (6 m/s constant velocity)

With respect to the Eqn (2) discussed, the pressure difference between inlet and outlet for both forward and reverse flow cases will take the form:

$$(P_{outlet} - P_{inlet}) = k_1(U^2) + k_2\left(\frac{\partial U}{\partial t}\right) + k_3(U).$$

Here, since this is steady state flow,  $k_2=0$ . Thus, equation will be of the form:

$$(P_{outlet} - P_{inlet}) = k_1(U^2) + k_3(U) \quad \text{Eq. 3}$$

The following two equations are solved for  $k_1$  and  $k_3$  for reverse flow:

$$((2^2) * k_1) + (2 * k_3) = 83.23 \quad \text{Eq. 4}$$

$$((6^2) * k_1) + (6 * k_3) = 741.8 \quad \text{Eq. 5}$$

Solving the two equations we get  $k_{1rev}=20.5$  and  $k_{3rev}=0.633$  for Reverse flow.

The following two equations are solved for  $k_1$  and  $k_3$  for forward flow:

$$((2^2) * k_1) + (2 * k_3) = 19.21 \quad \text{Eq. 6}$$

$$((6^2) * k_1) + (6 * k_3) = 169.843 \quad \text{Eq. 7}$$

Solving the two equations we get  $k_{1for} = 4.67$  and  $k_{3for} = 0.28$  for Forward flow.

These values of  $k_1$  and  $k_3$  can be verified by checking for the 4 m/sec steady state flow case, in Stage 3 of Case 3 (Ramped flow Case) – Refer Table 5.2 – at  $t = 7.5 \text{ sec}$ .

$$((4^2) * k_{1for}) + (4 * k_{3for}) = 75.84 \text{ m}^2/\text{sec}^2$$

is close to the observed value of  $75.98 \text{ m}^2/\text{sec}^2$  in Table 2 at  $t = 7.5 \text{ sec}$  for Forward Flow.

$$((4^2) * k_{1rev}) + (4 * k_{3rev}) = 330.53 \text{ m}^2/\text{sec}^2$$

is close to the observed value of  $331.85 \text{ m}^2/\text{sec}^2$  in Table 2 at  $t = 7.5 \text{ sec}$  for Reverse Flow.

For steady flow case, the constant  $k_1$  dominates for turbulent flow case. The ratio of

$(k_1)_R/(k_1)_F$  is 4.38 which has the dominant contribution to Diodicity.

When steady state velocities drop below 0.03 m/sec,  $\frac{(k_3)_{Rev}}{(k_3)_{For}}$  will begin to dominate – velocity 0.03 m/sec has been calculated equating  $k_1(U) = k_3(U)$  of equation (3) for Reverse flow case since that case gives the lower threshold value.

To validate the values of velocity at which Diodicity due to  $k_3$  dominates, let us refer to Fig 2 (b) of the research paper listed in Reference - [1], which studies how the diodicity of Tesla Valve changes with change in Reynold's flow regime from laminar to transitional flow regime.

In Fig 2(b) of the referred research paper, resistance offered by the Tesla Valve as measured for their Tesla Valve model is given for Forward and Reverse flow against the Reynold's number upto  $Re=2300$ , which is the start of transition from laminar to turbulent flow. The ratio of these resistances is diodicity – A dimensionless number, which is also plotted against Reynolds number in Fig 2(e) of the same research paper in logarithmic scale. However, we will refer to Fig 2(b) since it is in linear scale.

In this plot, for a Reynold's number of 2300 (Reynold's number for Transition regime), the diodicity comes out to be  $(Resistance_{(rev)}/Resistance_{(Forward)}) = 360/150 = 2.4$ .

For our case, Reynold's number of 2300 is achieved at around  $U=0.0108$  m/sec. For comparison, at 0.03 m/sec velocity where the dominance of  $k_3$  term begins for the Tesla Valve is  $Re=6382.97$ .

As per derived formula for solved values of  $k_1$  and  $k_3$  for forward and reverse flow,

Resistance<sub>(rev)</sub> for this velocity:

$$20.5 * (0.0108^2) + (0.633 * (0.0108)) = 9.22 \times 10^3$$

Resistance<sub>(forward)</sub> for this velocity:

$$4.67 * (0.0108^2) + (0.28 * (0.0108)) = 3.57 \times 10^3$$

Thus, giving a diodicity of 2.58, close to the diodicity achieved in the mentioned paper.

To take a second example, For  $Re = 2000$  of this plot, the diodicity comes out to be

$$\left( \frac{Resistance_{rev}}{Resistance_{Forward}} \right) = \frac{150}{80} = 2.4.$$

For our case, Reynold's number of 2000 is achieved at around  $U=0.0094$  m/sec.

Resistance<sub>(rev)</sub> for this velocity:

$$20.5 * (0.0094^2) + (0.633 * (0.0094)) = 7.7615 \times 10^3.$$

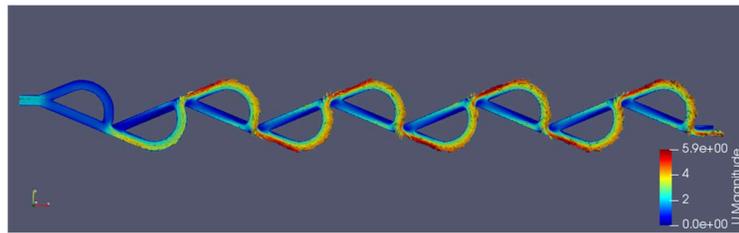
Resistance<sub>(forward)</sub> for this velocity:

$$4.67 * (0.0094^2) + (0.28 * (0.0094)) = 3.044 \times 10^3.$$

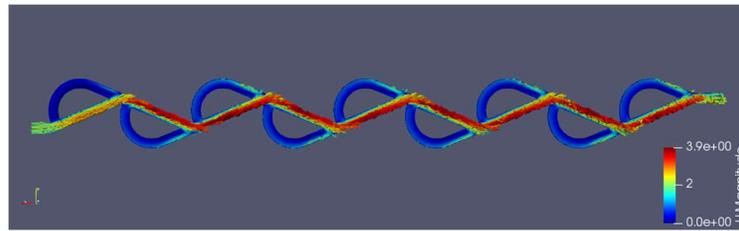
Thus, giving a diodicity of 2.54, close to the diodicity achieved in the mentioned paper.

The following section is devoted to the change in flow direction within the Tesla Valve channels as the flow transitions towards steady state – Refer Fig 5 and Fig 6.

Reverse flow for 2 m/s case and 6 m/s case are similar as seen in Fig 5 and 6. This is true for forward flow case as well. To confirm this, the passive scalar profiles for these cases were compared as shown in figure 7 and 8.

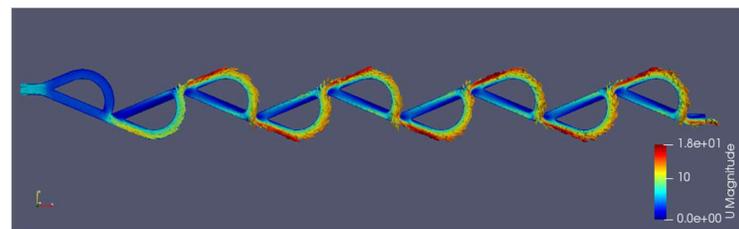


(a)

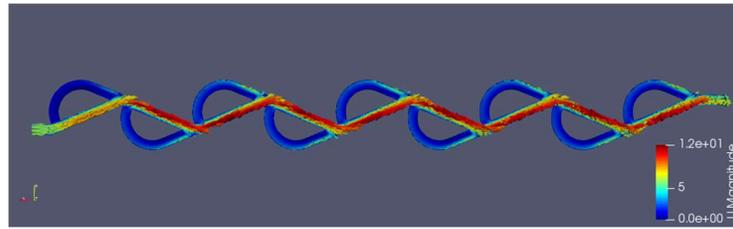


(b)

Fig 5 Velocity Profile for (a) reverse flow and (b) forward flow (ranging from 0 m/sec to 3.9 m/sec) for 2 m/sec inlet velocity



(a)



(b)

Fig 6 Velocity Profile for (a) reverse flow and (b) forward flow (ranging from 0 m/sec to 12 m/sec) for 6 m/sec inlet velocity

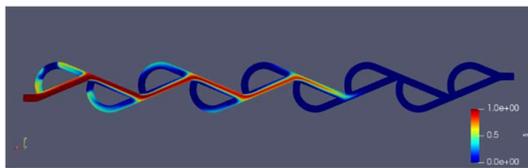
Figure 7 and Figure 8 compare the scalar profiles for forward and reverse flow respectively of the 2 m/s case on the left with the 6 m/s case on the right at the various stages of scalar flow. In the explanation given below, Stage 1 refers to when the scalar reaches 4<sup>th</sup> diodic separation (one-third of the flow path), stage 2 refers to when the scalar reaches 7<sup>th</sup> diodic separation (two-thirds of the flow path) and stage 3 refers to when the scalar reaches the outlet. As we can see, for all the three stages of flow, the scalar profiles are very similar for the 2 m/s and 6 m/s case, for both forward and reverse flow.



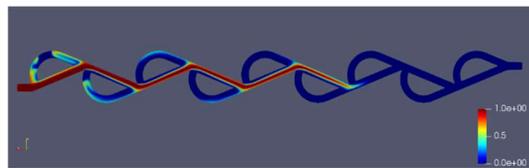
(a)



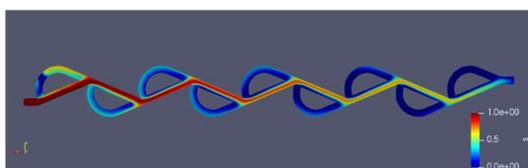
(b)



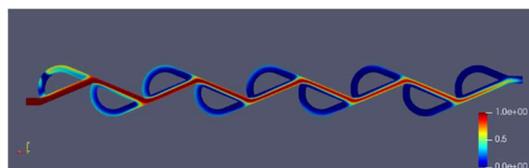
(c)



(d)



(e)



(f)

Fig. 7 (b): Scalar profiles for (a) forward flow of 2 m/sec when it reaches Stage 1: 4<sup>th</sup> diode separation, (b) forward flow of 6 m/sec when it reaches Stage 1: 4<sup>th</sup> diode separation, (c)

forward flow of 2 m/s case when it reaches Stage 2: 7<sup>th</sup> diode separation, (d) 6 m/s case when it reaches Stage 2: 7<sup>th</sup> diode separation, (e) forward flow of 2 m/sec when it reaches Stage 3: the outlet (Entry R), (f) forward flow of 6 m/sec when it reaches Stage 3: the outlet (Entry R)

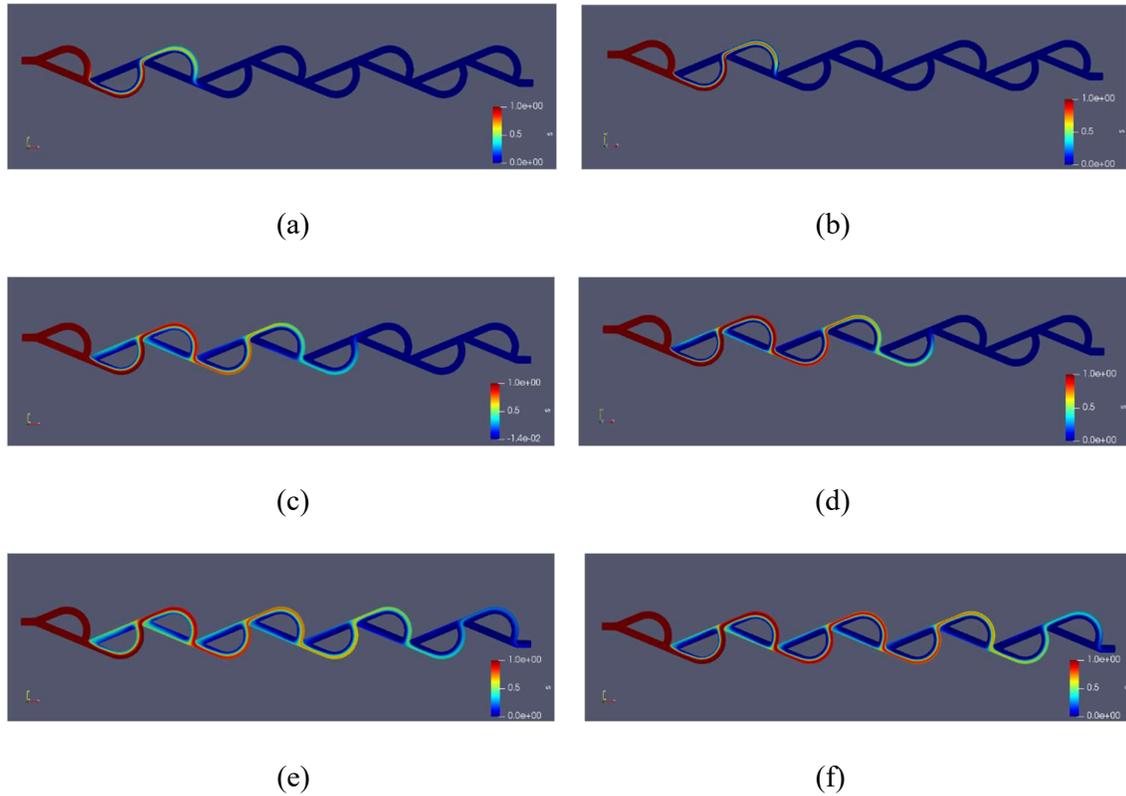


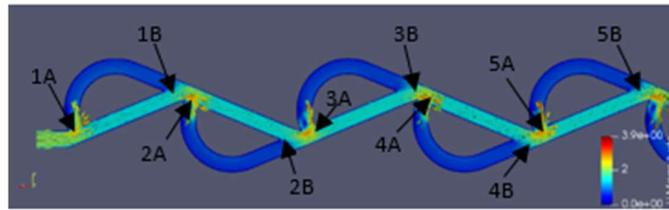
Fig. 8 Scalar profiles for reverse flow of (a) 2 m/s case when it reaches Stage 1: 4<sup>th</sup> diode separation, (b) 6 m/s case when it reaches Stage 1: 4<sup>th</sup> diode separation, (c) 2 m/s case when it reaches Stage 2: 7<sup>th</sup> diode separation, (d) 6 m/s case when it reaches Stage 2: 7<sup>th</sup> diode separation (e) 2 m/s case when it reaches Stage 3 outlet (Entry F) and (f) 6 m/s case when it reaches Stage 3 outlet (Entry F)

For the 2 m/s forward flow case, Stage 1 was reached in 1.15 sec, Stage 2 in 1.9 sec and Stage 3 in 2.7 sec. For 6 m/s forward flow case, Stage 1 was reached in 0.38 sec, Stage 2 in 0.63 sec and Stage 3 in 0.89 sec. The time taken for the passive scalar to reach each stage is directly proportional to velocity of flow.

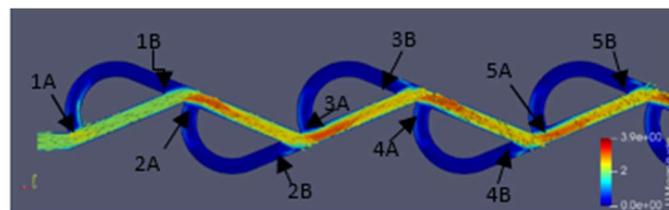
The same can be said for the reverse flow case - for the 2 m/s reverse flow case, Stage 1 was reached in 1.37 sec, Stage 2 in 2.19 sec and Stage 3 in 3.04 sec. For 6 m/s reverse flow case, Stage 1 was reached in 0.46 sec, Stage 2 in 0.73 sec and Stage 3 in 1.01 sec.

A close up of the velocity profiles for forward and reverse flow has been shown for Fig 9 and Fig 10 for the 2 m/s inlet velocity case, to further understand how flow behaviour changes as it develops. Since the velocity profile is similar for the 2 m/s and 6 m/s case (as established above), 2 m/s case is sufficient to understand how the velocity profile changes as it develops.

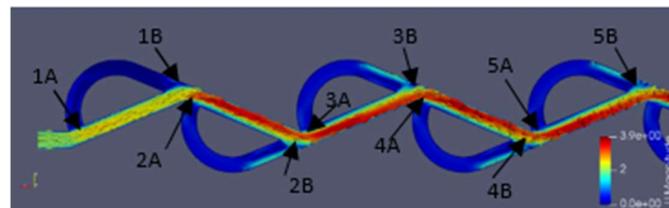
The velocity profile closeups for 2 m/s inlet velocity in figures 9 and 10 show how the vector direction of velocity changes as the flow develops. In each of the figures, in the rounded portion of the diodic separation, the straight entrance has been labelled as nB and the curved entrance has been labelled as nA (n referring to the diodic separation as stated in Figure 1).



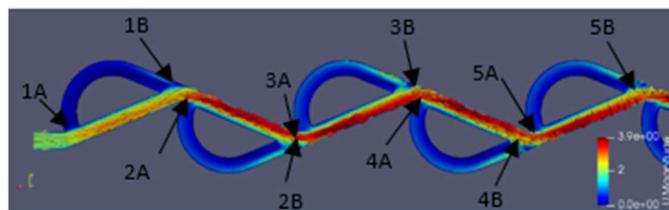
(a)



(b)

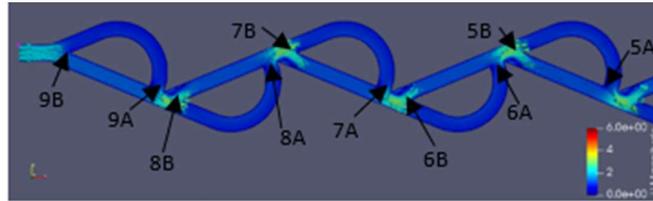


(c)

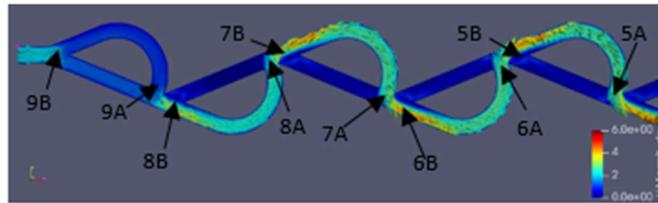


(d)

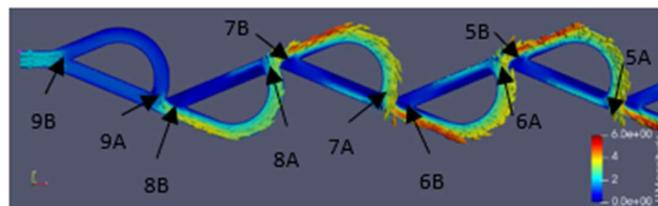
Fig 9: Velocity profiles for 2 m/s inlet velocity (forward flow): a) After 0.1 seconds b) After 0.5 seconds c) After 1 second d) After 5 seconds.



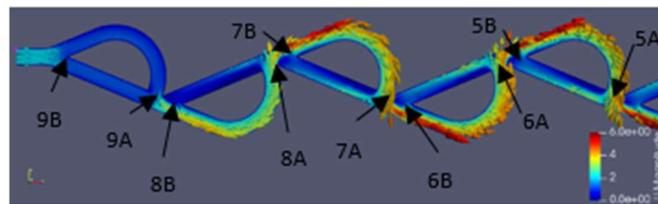
(a)



(b)



(c)



(d)

Fig 10: Velocity profiles for 2 m/s inlet velocity (reverse flow) from top left to right in increasing order of time from the start of simulation: a) After 0.1 seconds b) After 0.5 seconds c) After 1 second d) After 5 seconds.

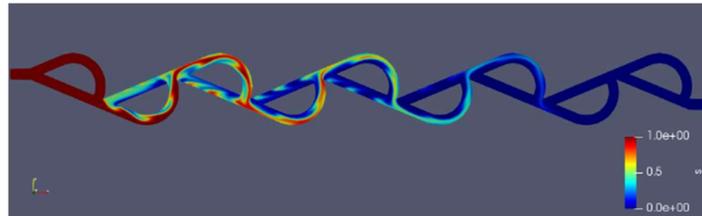
In the forward flow case, as the flow develops, we can see that at all the stages of the fluid flow development, the velocity of fluid in the straight channel is higher than in the rounded portion of the Tesla Valve. In the rounded portion, initially, the fluid flows in the forward direction (from n A to nB) as seen in Figure 9 (a), slowly decreases in magnitude in the

forward direction as seen in 9 (b) and 9 (c) and then reverses in course as it moves from nB to nA (Figure 9 (b)). Since, as the flow develops, the fluid flow in the straight channel increases, the flow velocity at nA for the rounded portion in the direction of nA to nB reduces. At the same time, since the velocity in the straight channel increases, there is impingement and subsequent backflow at nB, resulting in reversal of direction of velocity in the rounded portion (initial direction nA to nB, final direction nB to nA).

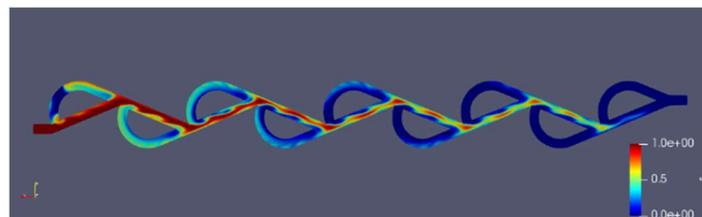
In the reverse flow case, as the flow develops, the velocity of the fluid in the rounded channel becomes higher as compared to the straight channel. The fluid in the straight channel can be seen to reverse its direction of flow (from nB towards nA to nA towards nB) from entrance 8B onwards as seen in figure 10 (b), (c) and (d). In the straight and rounded channel portion 9B to 9A, the direction of flow remains the same throughout the simulation, since the fluid flow is coming from the inlet directly to these channels. The reason for reversal of flow from 8B onwards, as the flow develops is that the velocity of fluid in the straight channel in the nB to nA direction decreases near the nB region. At the same time, due to increase in flow velocity in the rounded portion, the fluid impinges at the straight walls and splatters in both directions. Thus the velocity direction reverses to nA to nB as the flow develops.

Case 2: Pulsating flow  $5 + 1\sin(40\pi t)$  m/s for forward and reverse flow in Tesla Valve

Since the flow follows a pulsating pattern, the passive scalar in the Tesla Valve develops a pulsed profile as well. Refer to Figure 11 for passive scalar profile for both forward and reverse flow case, taken from 1 second after simulation. The inlet velocity is at the mean value of 5 m/sec for this case.



(a)



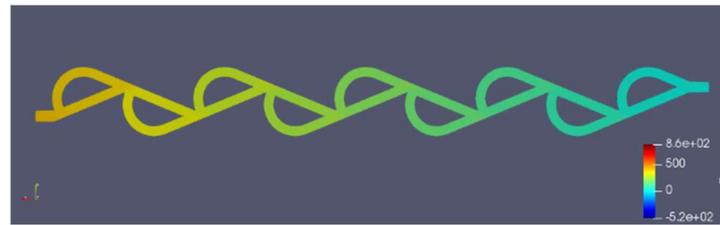
(b)

Figure 11: Scalar profile for pulsating flow rate for a) reverse and b) forward flow through Tesla Valve.

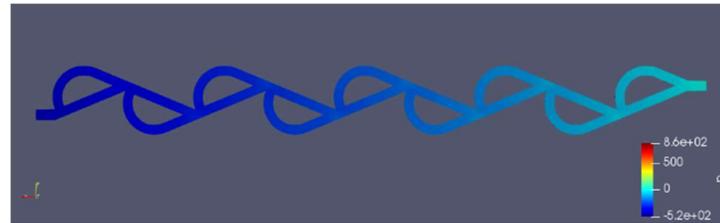
It was observed that after 1 second of flow simulation, in both forward and reverse cases, the pressure profile also changed in a periodic, oscillatory fashion and continued till the end of simulation (6 seconds). Figure 12 depicts the variation in pressure profile through 1 cycle of sinusoidal velocity change, from 1.00 sec to 1.05 sec.



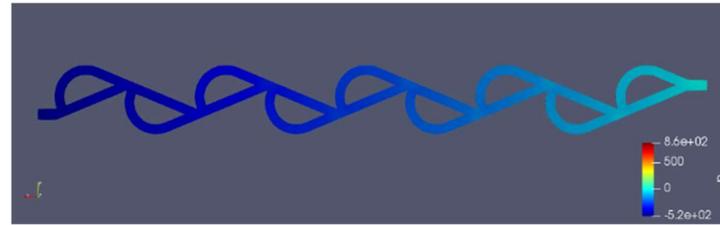
(a)



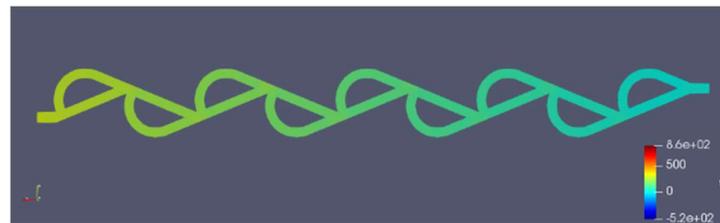
(b)



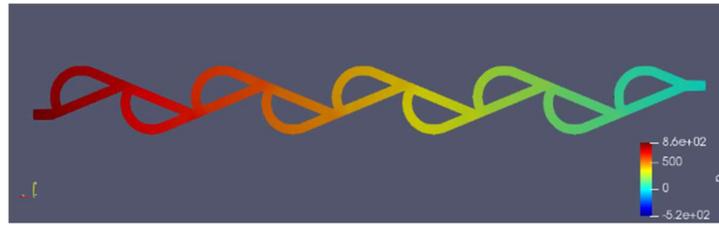
(c)



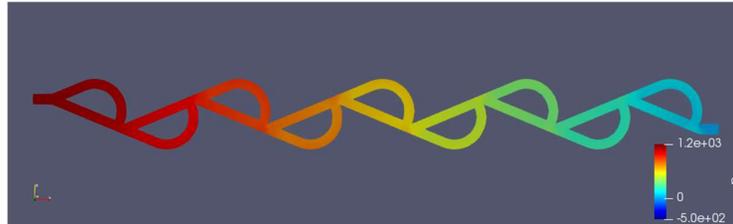
(d)



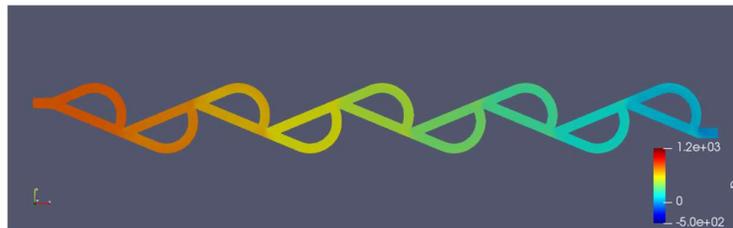
(e)



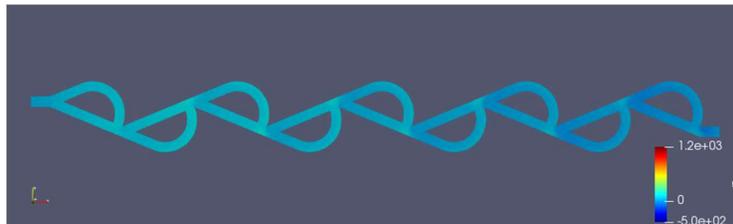
(f)



(g)



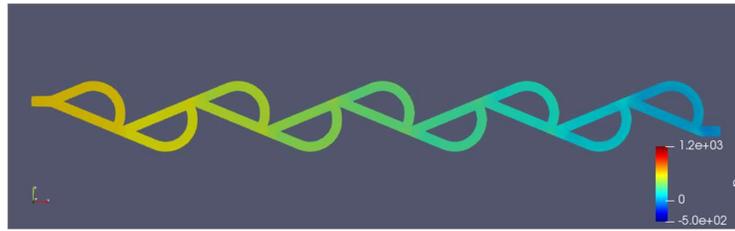
(h)



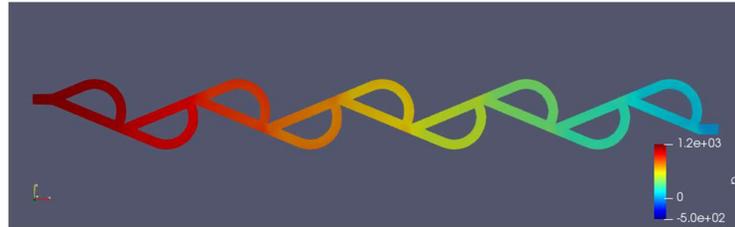
(i)



(j)



(k)



(l)

Figure 12: Pressure variation for forward flow after (a) 1.00 sec, (b) 1.01 sec, (c) 1.02 sec, (d) 1.03 sec, (e) 1.04 sec, (f) 1.05 sec and Pressure Variation for reverse flow after (g) 1.00 sec, (h) 1.01 sec, (i) 1.02 sec, (j) 1.03 sec, (k) 1.04 sec, (l) 1.05 sec

It was also observed that due to a pulsed velocity profile, the pressure profile also changes periodically for both forward and reverse flow case. The pressure at the outlet was kept at  $0 \text{ kg}^2/\text{sec}^2$  as stated in Section 4.2 (initial and boundary conditions), while the inlet had zero – gradient boundary condition to allow pressure development. As can be seen, the pressure at the inlet (measured relative to the  $0 \text{ kg}^2/\text{sec}^2$  value fixed at the outlet), also changes periodically, decreasing through 1.00 sec to 1.03 sec, and then increasing back through 1.04 and 1.05 seconds.

Following the discussion regarding the nature of changing pressure and scalar profile for pulsating flow, a detailed analysis is performed on the variation in pressure difference and velocity after the flow reaches periodic, oscillating state in the Tesla Valve. The pressure difference between the inlet and the outlet, and its variation with pulsating flow have been studied in detail for the time period of 1.00 sec to 1.25 sec as shown in Figure 13, by taking a probe at the inlet end for the two types of cases. The probe for reverse flow case was taken at  $[0,0.05,0.00051]$  - near the inlet center for reverse flow, and the probe for forward flow case was taken at  $[7.025,-0.25,0.000579]$  - near the inlet center for forward flow.

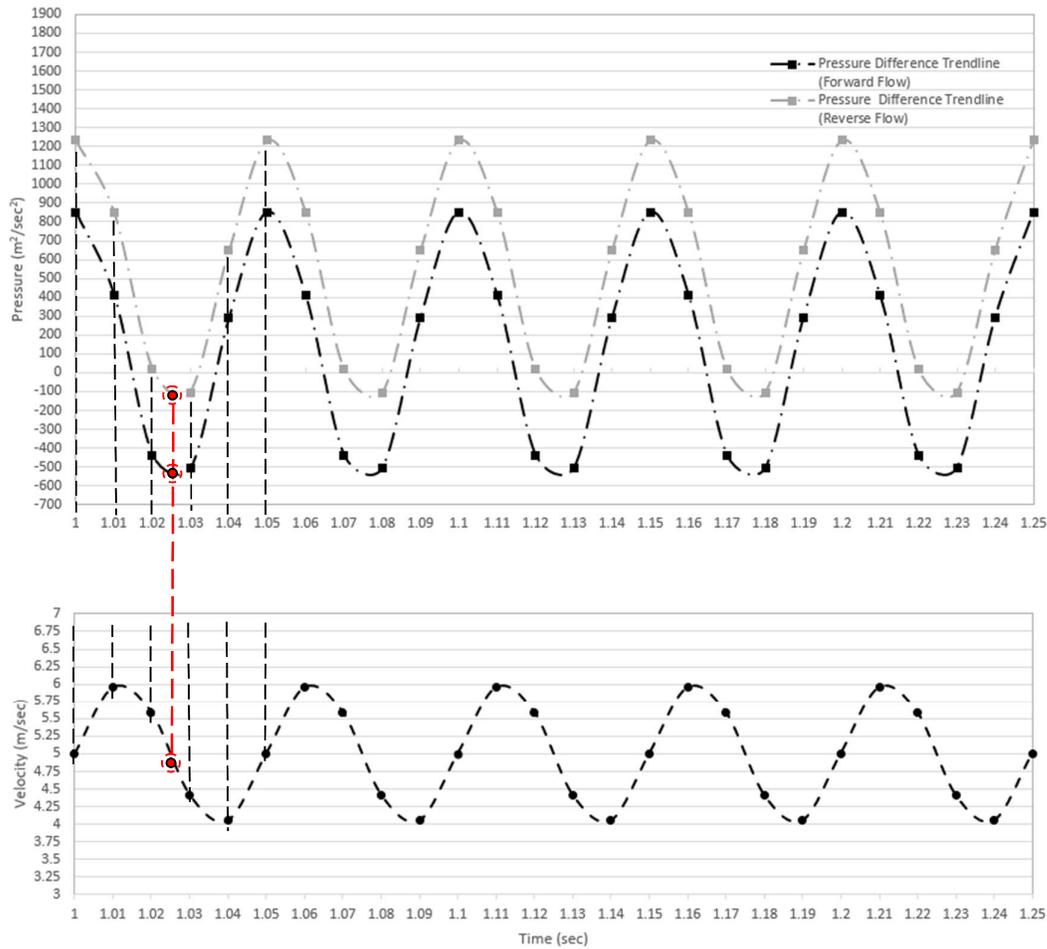


Fig 13: Pressure difference and Velocity plots - pulsating flow for Forward and Reverse Flow through Tesla Valve from 1.00 sec to 1.25 sec.

As it can be seen, the pressure difference for both the forward and reverse flow both follow a sinusoidal trendline, similar to velocity flow. However, both have a phase difference of 90 degrees against the pulsating velocity graph. The highest pressure difference values are reached near the mean velocity value of 5 m/sec when the velocity is increasing. The lowest pressure difference values are reached at the mean velocity value of 5 m/sec when the velocity is decreasing. (This has been marked in red in the graph as it is an interpolated value – occurs at 1.025 sec). The lowest pressure difference values are negative, which means that the pressure at the inlet is less than the pressure at the outlet.

From the above observations, we can conclude that the peaks and troughs of the pressure trough are occurring at 5 m/sec, when the rate of change of velocity is at its highest. The

change in pressure difference for both forward and reverse flow with time thus seems directly proportional to the rate of change of velocity.

Table 5.1: Velocity and corresponding pressure difference for forward and reverse flow from 1.00 sec to 1.05 sec

Time (sec)	Velocity (m/sec)	Pressure Difference Reverse Flow ( $\text{m}^2/\text{sec}^2$ )	Pressure Difference Forward Flow ( $\text{m}^2/\text{sec}^2$ )	Variation between Pressure Difference of Forward versus Reverse Flow ( $\text{m}^2/\text{sec}^2$ )
1.00	5	1232.33	850.247	382.083
1.01	5.951	848.418	480.115	440.303
1.02	5.588	18.4283	-437.225	455.653
1.03	4.412	-105.147	-504.971	399.824
1.04	4.049	647.951	292.656	355.295
1.05	5	1232.69	851.029	381.661

Table 5.1 denotes variation in inlet velocity and pressure difference for forward and reverse flow from 1.00 seconds to 1.05 seconds. It was observed that these values continued periodically for pressure differences for both Forward and Reverse Flow in the time period of 0.05 seconds throughout the simulation from  $t = 1 \text{ second}$  to  $t = 6 \text{ seconds}$ .

An interesting point to note is that the pressure difference between the forward flow case and the reverse flow case has very low variation, as can be seen in Table 5.1 – varying from a value of 355  $\text{m}^2/\text{sec}^2$  to 455  $\text{m}^2/\text{sec}^2$ . This is interesting because, if one end of the valve in both forward and reverse flow case has a fixed pressure value, the variation in pressure difference between the other two junctions remains fairly constant. Thus, suppose if two tesla valves at forward and reverse state are connected at a common junction, with the purpose of developing a pressure difference between the other two ends. Then, any variation or fluctuation in pressure at the common junction will be dampened and will have a small effect on the developed pressure differential, thus acting as a pressure dampener or filter and keeping a near constant pressure difference between the other two ends.

To gain an understanding of the sinusoidal pressure variation at the inlet, a formula for pressure variation needs to be derived using Equation 2.

As per Eqn 2, the Pressure Difference between Inlet and outlet will take the following form:

$$(P_{outlet} - P_{inlet}) = k_1((5 + 1\sin(40\pi t))^2) + k_2\left(\frac{\partial(5+1\sin(40\pi t))}{\partial t}\right) + k_3(5 + 1\sin(40\pi t)) \quad Eq.8$$

Solving the derivative for the second term on Right Hand Side, this gives us,

$$(P_{outlet} - P_{inlet}) = k_1((5 + 1\sin(40\pi t))^2) + k_2(40\pi \cos(40\pi t)) + k_3(5 + 1\sin(40\pi t)) \quad Eq.9$$

$k_1$ ,  $k_2$ , and  $k_3$  take different values for forward and reverse flow, as had been observed in the steady state flow cases.

For the forward and reverse flow case in pulsating flow, three of the five values tabulated in Table 5.1 will be used to derive the constants  $k_1$ ,  $k_2$ , and  $k_3$ . The other two values will be used to verify if the formula used is correct.

First, we consider Reverse flow case:

Inputting values into Eqn (9) for  $t=1.00$  sec from Table 1, we get

$$1232.33 = 25 k_{1Rev} + 125.66 k_{2Rev} + 5 k_{3Rev} \quad Eq.10$$

Inputting values into Eqn (9) for  $t=1.01$  sec from Table 1,

$$848.418 = 35.41 k_{1Rev} + 38.83 k_{2Rev} + 5.951 k_{3Rev} \quad Eq.11$$

Inputting values into Eqn (9) for  $t=1.04$  sec from Table 1,

$$647.951 = 16.38 k_{1Rev} + 38.83 k_{2Rev} + 4.048 k_{3Rev} \quad Eq.12$$

Solving the three equations we get  $k_{1Rev} = -0.2134$ ,  $k_{2Rev} = 5.5727$ ,  $k_{3Rev} = 107.476$

Thus giving the following equation for reverse flow:

$$(P_{outlet} - P_{inlet}) = (-0.21 * (5 + 1\sin(40\pi t))^2) + 5.57 * (40\pi \cos(40\pi t)) + 107.47 * (5 + 1\sin(40\pi t)) \quad Eq.13$$

To verify the above equation, it will be checked against  $t = 1.02$  sec and  $t = 1.03$  sec:

For  $t = 1.02$  sec,

$$(P_{outlet} - P_{inlet})_{check_{rev}} = (-0.21 * (5 + 1\sin(40\pi * 1.02))^2) + 5.57 * (40\pi \cos(40\pi * 1.02)) + 107.47 * (5 + 1\sin(40\pi * 1.02))$$

Giving  $\Delta P_{check_{rev}}$  as  $27.7 \text{ m}^2/\text{sec}^2$ , close to observed value of  $18.42 \text{ m}^2/\text{sec}^2$ .

For  $t=1.03$  sec,

$$(P_{outlet} - P_{inlet})_{check_{rev}} = (-0.21 * (5 + 1\sin(40\pi * 1.03))^2) + 5.57 * (40\pi \cos(40\pi * 1.03)) + 107.47 * (5 + 1\sin(40\pi * 1.03))$$

Giving  $\Delta P_{check_{rev}}$  as  $-96.19 \text{ m}^2/\text{sec}^2$ , close to observed value of  $-105.147 \text{ m}^2/\text{sec}^2$ .

Now considering Forward flow case:

Inputting values into Eqn (9) for  $t = 1.00$  sec from Table 1, we get

$$850.247 = 25 k_{1For} + 125.66 k_{2For} + 5 k_{3For} \quad Eq.14$$

Inputting values into Eqn (9) for  $t = 1.01$  sec from Table 1,

$$408.115 = 35.41 k_{1For} + 38.83 k_{2For} + 5.951 k_{3For} \quad Eq.15$$

Inputting values into Eqn (9) for  $t = 1.04$  sec from Table 1,

$$292.656 = 16.38 k_{1For} + 38.83 k_{2For} + 4.048 k_{3For} \quad Eq.16$$

Solving the three equations we get  $k_{1For} = 7.46, k_{2For} = 5.83, k_{3For} = -13.872$

Thus giving the following equation for reverse flow:

$$(P_{outlet} - P_{inlet}) = (7.46 * (5 + 1 \sin(40\pi t))^2) + 5.83 * (40\pi \cos(40\pi t)) - 13.872 * (5 + 1 \sin(40\pi t)) \quad Eq.17$$

To verify the above equation, it will be checked against  $t=1.02$  sec and  $t=1.03$  sec:

$$\begin{aligned} \text{For } t = 1.02 \text{ sec, } (P_{outlet} - P_{inlet})_{check(for)} = \\ (7.46 * (5 + 1 \sin(40\pi * 1.02))^2) + 5.83 * (40\pi \cos(40\pi * 1.02)) \\ - 13.872 * (5 + 1 \sin(40\pi * 1.02)) \end{aligned}$$

Giving  $\Delta P_{check(for)}$  as  $-437.266 \text{ m}^2/\text{sec}^2$ , close to observed value of  $-437.225 \text{ m}^2/\text{sec}^2$ .

$$\begin{aligned} \text{For } t = 1.03 \text{ sec, } (P_{outlet} - P_{inlet})_{check(for)} = (7.46 * (5 + 1 \sin(40\pi * 1.03))^2) + \\ 5.83 * (40\pi \cos(40\pi * 1.03)) - 13.872 * (5 + 1 \sin(40\pi * 1.03)) \end{aligned}$$

Giving  $\Delta P_{check(for)}$  as  $-508.672 \text{ m}^2/\text{sec}^2$ , close to observed value of  $-504.971 \text{ m}^2/\text{sec}^2$ .

Thus the constants  $k_1, k_2, k_3$  for forward and reverse flow have been calculated and verified as follows:

$$k_{1Rev} = -0.2134, k_{2Rev} = 5.5727, k_{3Rev} = 107.476$$

$$k_{1For} = 7.46, k_{2For} = 5.83, k_{3For} = -13.872$$

For Forward and Reverse flows in pulsating flows, it can be observed that  $k_1$  and  $k_3$  are greater than  $k_2$ . However, it should also be noted that  $k_2$  is multiplied with a frequency term ( $40 * 3.1415$ ) which came into equation 9 as a result of derivative of velocity. Thus, the term containing  $k_2$  dominates for velocity pulsating about 5 m/sec mean velocity.

Dominance of the term containing  $k_2$  explains why the pressure curves seem to have a cosine curve in Figure 13 for both forward and reverse flow – the term containing  $k_2$  in Equations 13 and 17 have a cosine term which dominates for the respective flow.

The term containing  $k_2$  dominates and defines the phase change for Forward and Reverse flows thus contributing to the 90 degree phase difference with respect to velocity.

It is observed that there is very little variation in pressure difference between Forward and Reverse flow, as seen in Table 5.1.

This is explained by the following:

- 1) In terms of the difference between pressure for forward and reverse flow at a particular time step, the difference between  $k_{2Rev}$  and  $k_{2For}$  is small,  $k_{2For}$  being only 5 % of  $k_{2rev}$ .
- 2) Hence, the first and the third terms,  $k_1$  and  $k_3$  would dominate. Since the fluctuation of velocity is 1 m/sec and the mean velocity about which fluctuation occurs is 5 m/sec ( $U = (5 + 1\sin(40\pi t))$ ), there would be very little variation about the mean for the pressure value as well.

To clarify this, let us calculate the mean value of Variation between Pressure Difference of Forward versus Reverse Flow ( $m^2 /sec^2$ ) tabulated in Table 5.1:  $402 m^2/sec^2$ .

For the mean value of  $U = (5 \text{ m/sec})$ , considering only the terms containing  $k_1$  and  $k_3$  for forward and reverse flow, we get,

$$(P_{(outlet)} - P_{(inlet)})_{for} = 7.46(5^2) - 13.872 (5) = 117.14$$

$$(P_{outlet} - P_{inlet})_{rev} = -0.2134(5^2) + 107.476 (5) = 532.045$$

Giving,  $\Delta P = 414.905 m^2/sec^2$ .

which is close to the mean value calculated.

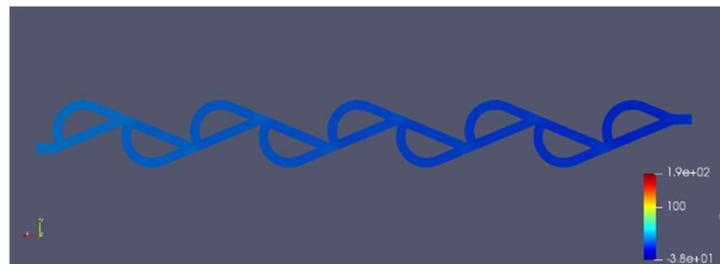
Since both forward and Reverse flows will follow the same phase (both change as a function of cosine), for high frequency pulsating flows, the pressure difference will have little variation about the mean. This can be ascertained from the data in Table 5.1, where the pressure difference values in the Variation between Pressure Difference of Forward versus Reverse Flow ( $m^2 /sec^2$ ) change by about 13.5 % about the mean value. This explains and justifies that for two tesla valves at forward and reverse state connected at a common junction, any small, high frequency fluctuations in velocity, about its mean value at the common junction will be dampened and will have a small effect on the developed pressure differential.

### Case 3: Ramped flow in forward and reverse flow through Tesla Valve

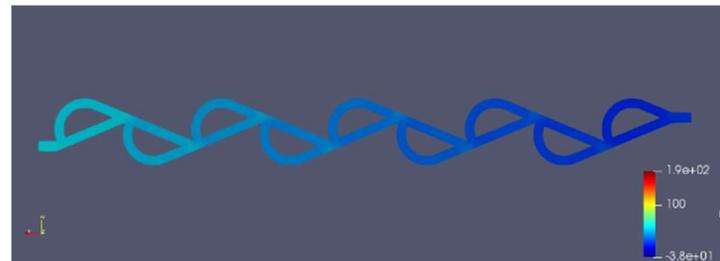
To further understand the effect of increasing flow rate on pressure difference and diodicity, the next case was for varying ramped flow. The following stages were simulated:

- 1) From 0 to 2.5 secs, the inlet velocity was constant at 2 m/s (Simulation begins)
- 2) From 2.5 to 5 secs, the inlet velocity was ramped from 2 m/s to 4 m/s
- 3) From 5 to 7.5 secs, the inlet velocity was constant 4 m/s
- 4) From 7.5 to 8.75 secs, the inlet velocity was ramped from 4 m/s to 6 m/s
- 5) From 8.75 to 12.5 secs, the fluid flow rate was constant at 6 m/s. (End of Simulation).

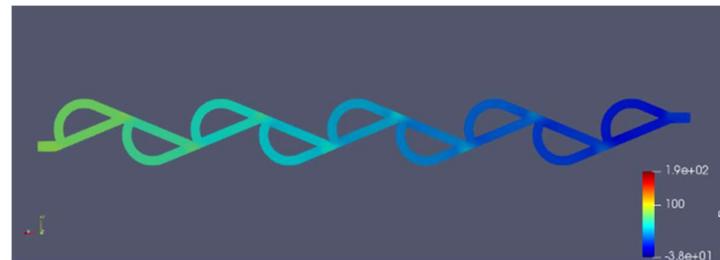
The resulting pressure differences during constant inlet flow as well as ramped flow for forward and reverse flow are shown in Figures 14 and 15.



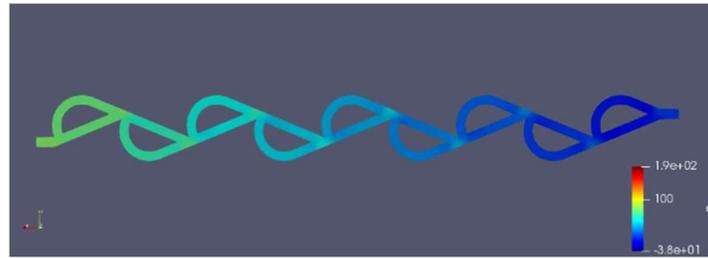
(a)



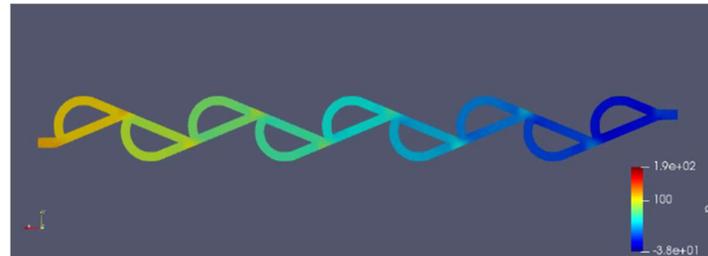
(b)



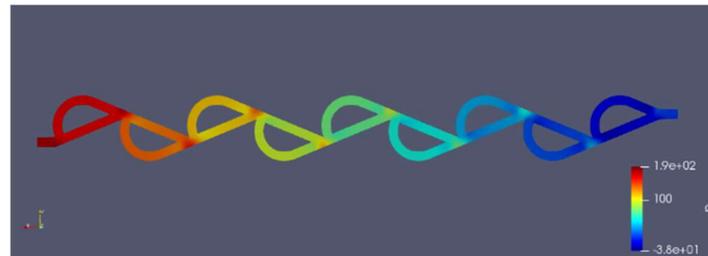
(c)



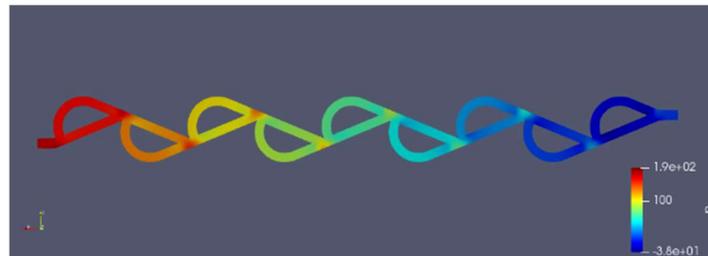
(d)



(e)

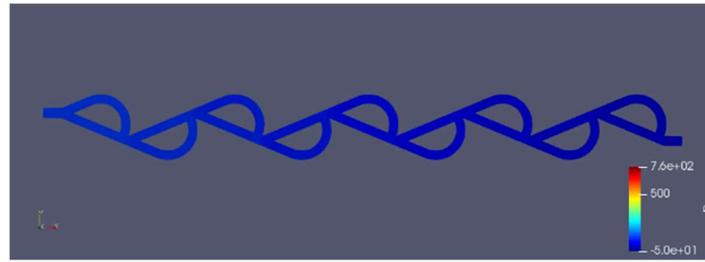


(f)

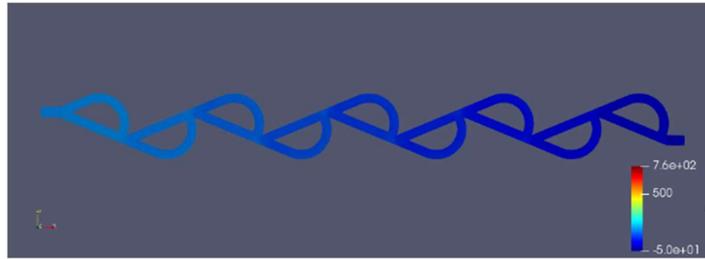


(g)

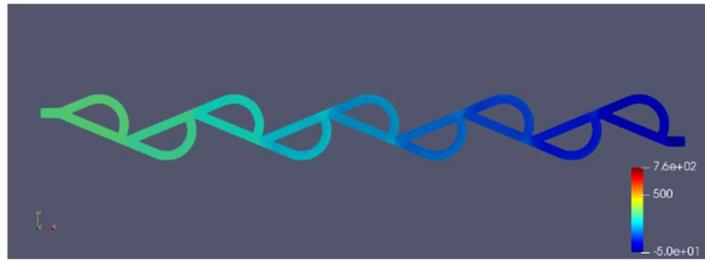
Figure 14: Pressure profile for forward flow at (a) 2.5 sec, (b) 3.5 sec, (c) 5 sec, (d) 7.5 sec, (e) 8 sec, (f) 8.75 sec and (g) 12.5 sec



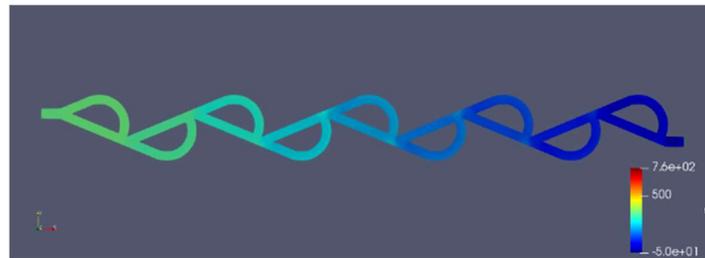
(a)



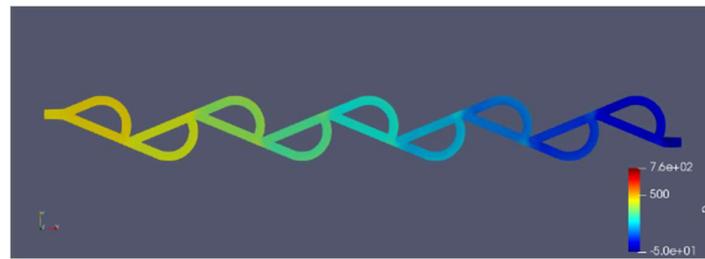
(b)



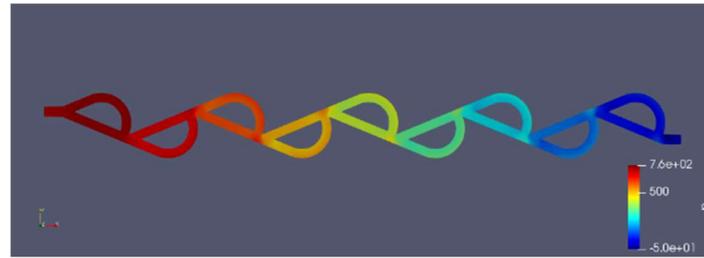
(c)



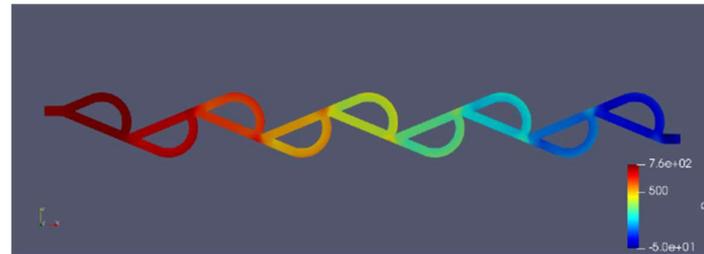
(d)



(e)



(f)



(g)

Figure 15: Pressure profile for reverse flow at (a) 2.5 sec, (b) 3.5 sec, (c) 5 sec, (d) 7.5 sec, (e) 8 sec, (f) 8.75 sec and (g) 12.5 sec

The Figures 14 and 15 show the pressure profiles at the end of each stage, as well as two cases during ramping (Fig 14 and 15 (b) and (e)). As we can see, for both forward and reverse flow, the pressure at the inlet keeps increasing with increase in velocity. To understand this further, the pressure differences and velocity were plotted from 0 seconds to 12.5 seconds.

The pressure values near the inlet for forward and reverse flow case were taken using a probe point. The probe for reverse flow case was taken at  $[0,0.05,0.00051]$  - near the inlet center for reverse flow, and the probe for forward flow case was taken at  $[7.025,-0.25,0.000579]$  - near the inlet center for forward flow.

It was found that the pressure at the inlet end for both the forward and reverse flow case got ramped up along with velocity. This, is in line with the observation in the previous case, where it was found that the change in velocity results in change in pressure for both forward and reverse flow, as a result of which the diodicity gets affected.

From the results, it can be inferred that the pressure variation with time, is directly proportional to velocity variation with time. The ramping rate of velocity in stage 4 is twice as much as the ramping rate in stage 2.

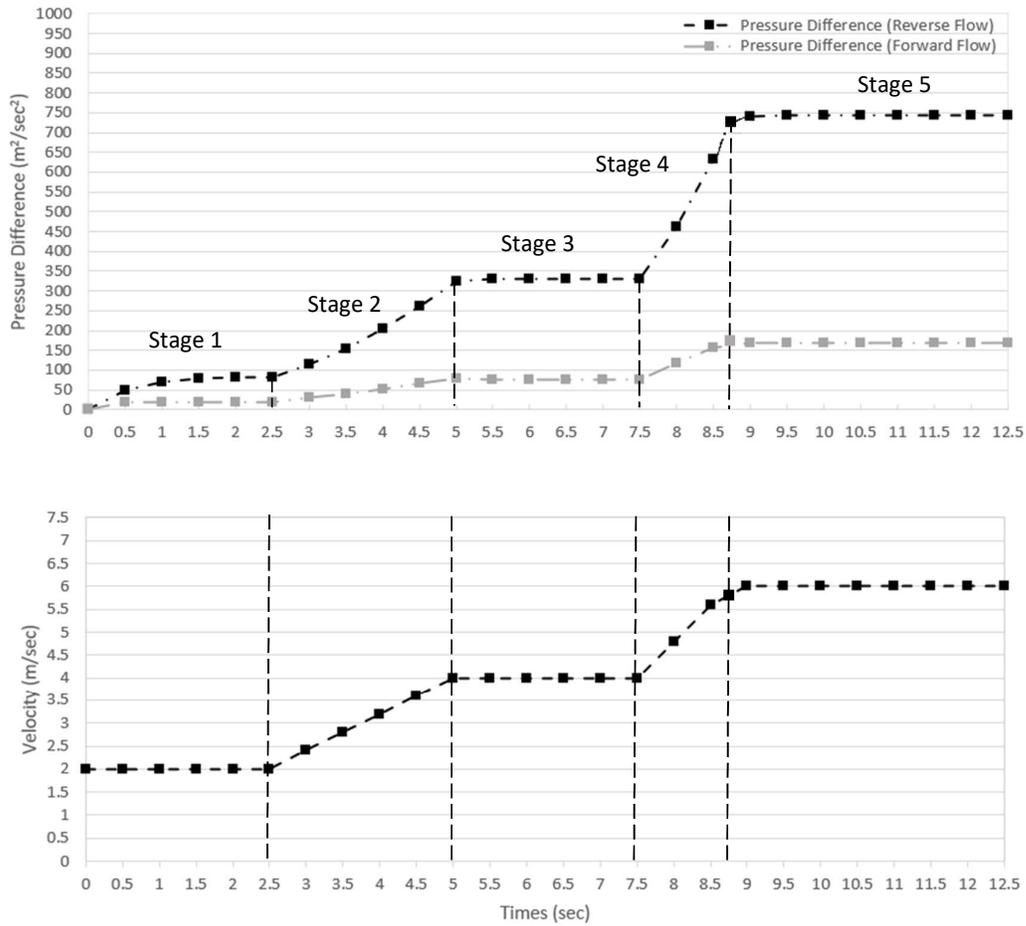


Fig 16: Pressure difference and Velocity plots - ramped flow for Forward and Reverse Flow through Tesla Valve from 0 sec to 12.5 sec.

Table 5.2: Pressure difference for Forward and Reverse Flow at the end of each stage of Ramping.

Time (sec)	Velocity (m/sec)	Pressure Difference Reverse Flow ( $m^2/sec^2$ )	Pressure Difference Forward Flow ( $m^2/sec^2$ )	Ratio of Pressure Differences (Diodicity)	Variation in Pressure Difference of Forward versus Reverse Flow ( $m^2/sec^2$ )
2.50	2	83.24	19.21	4.33	64.03
5.00	4	323.90	79.94	4.05	243.96
7.50	4	331.85	75.98	4.36	255.87
8.75	6	728.21	177.85	4.09	550.36

Time (sec)	Velocity (m/sec)	Pressure Difference Reverse Flow (m <sup>2</sup> /sec <sup>2</sup> )	Pressure Difference Forward Flow (m <sup>2</sup> /sec <sup>2</sup> )	Ratio of Pressure Differences (Diodicity)	Variation in Pressure Difference of Forward versus Reverse Flow (m <sup>2</sup> /sec <sup>2</sup> )
12.5	6	744.06	169.84	4.38	574.22

As done in Case 1 and Case 2, to gain an understanding of the sinusoidal pressure variation at the inlet, a formula for pressure variation needs to be derived using Equation 2.

As per Eqn 2, the Pressure Difference between Inlet and outlet will take the following form:

$$(P_{outlet} - P_{inlet}) = k_1((U^2) + k_2\left(\frac{\partial(U)}{\partial t}\right) + k_3(U)) \quad Eq.18$$

Where  $\frac{\partial(U)}{\partial t}$  will be given by the ramping value –  $\Delta U/\Delta t$  (ramping for Stage 2 is 0.8 m/sec<sup>2</sup> while it is 1.6 m/sec<sup>2</sup> for Stage 4).

For the forward and reverse flow case in ramped flow, three of the four values charted in Figure 16 in the stage of ramping in Stage 2 will be used to derive the constants  $k_1$ ,  $k_2$ , and  $k_3$ . The fourth value in Stage 2 and the two values in the stage of ramping in Stage 4 will be used to verify the values of  $k_1$ ,  $k_2$ , and  $k_3$ . Note that Stage 4 ramping is at a different rate than the Stage 2 ramping, but the constants  $k_1$ ,  $k_2$ , and  $k_3$  remain the same for each of Forward and Reverse flow case. Refer to Table 5.3 for the exact observed values at these time steps.

Table 5.3: Observed values of Pressure difference for Forward and Reverse Flow for verification of  $k_1$ ,  $k_2$ , and  $k_3$  constants

Time (sec)	Velocity (m/sec)	Pressure Difference Reverse Flow (m <sup>2</sup> /sec <sup>2</sup> )	Pressure Difference Forward Flow (m <sup>2</sup> /sec <sup>2</sup> )	Inlet Velocity Ramping (m/sec <sup>2</sup> )
3.00	2.4	114.866	31.8313	0.8
8.00	4.8	462.358	117.176	0.8
8.50	5.6	632.569	156.095	1.6

Considering Forward Flow Case,

At 3.5 seconds, inputting  $U = 2.8 \text{ m/sec}$  and  $\frac{\partial(U)}{\partial t}=0.8 \text{ m/sec}^2$  in Eqn (18)

$$41.35 = k_{1_{For}} (2.8^2) + k_{2_{For}} (0.8) + k_{3_{For}} (2.8) \quad Eq.19$$

At 4 seconds, inputting  $U=3.2 \text{ m/sec}$  and  $\partial(U)/\partial t=0.8 \text{ m/sec}^2$  in Eqn (18)

$$52.68 = k_{1_{For}} (3.2^2) + k_{2_{For}} (0.8) + k_{3_{For}} (3.2) \quad Eq.20$$

At 4.5 seconds, inputting  $U=3.6 \text{ m/sec}$  and  $\frac{\partial(U)}{\partial t}=0.8 \text{ m/sec}^2$  in Eqn (18)

$$65.58 = k_{1_{For}} (3.6^2) + k_{2_{For}} (0.8) + k_{3_{For}} (3.6) \quad Eq.21$$

Thus, the following formula is followed for Pressure Difference between inlet and outlet for Forward Flow Case:

$$(P_{outlet} - P_{inlet})_{For} = 4.928((U^2) + (7.824 * \frac{\Delta U}{\Delta t}) - 1.2655(U)) \quad Eq.22$$

Verifying the Eqn (22) at  $t = 3 \text{ seconds}$ , where  $U$  is at  $2.4 \text{ m/sec}$ , and

$$\frac{\Delta U}{\Delta t} = 0.8 \text{ m/sec}^2$$

$(P_{outlet} - P_{inlet})_{For} = 31.6 \text{ m}^2/\text{sec}^2$  which is close to the observed value of  $31.8313 \text{ m}^2/\text{sec}^2$ .

Verifying the Eqn (22) at  $t = 8 \text{ seconds}$ , where  $U$  is at  $4.8 \text{ m/sec}$ , and  $\frac{\Delta U}{\Delta t} = 1.6 \text{ m/sec}^2$

$(P_{outlet} - P_{inlet})_{For} = 120 \text{ m}^2/\text{sec}^2$  which is close to the observed value of  $117.176 \text{ m}^2/\text{sec}^2$  .

Verifying the Eqn (22) at  $t = 8.5 \text{ seconds}$ , where  $U$  is at  $5.6 \text{ m/sec}$ , and  $\frac{\Delta U}{\Delta t} = 1.6 \text{ m/sec}^2$

$(P_{outlet} - P_{inlet})_{For} = 160 \text{ m}^2/\text{sec}^2$  which is close to the observed value of  $156.095 \text{ m}^2/\text{sec}^2$ .

Considering Reverse Flow Case,

At 3.5 seconds, inputting  $U=2.8 \text{ m/sec}$  and  $\frac{\partial(U)}{\partial t} = 0.8 \text{ m/sec}^2$  in Eqn (18)

$$155.221 = k_{1Rev} (2.8^2) + k_{2Rev} (0.8) + k_{3Rev} (2.8) \quad \text{Eq.23}$$

At 4 seconds, inputting  $U=3.2 \text{ m/sec}$  and  $\partial(U)/\partial t=0.8 \text{ m/sec}^2$  in Eqn (18)

$$204.57 = k_{1Rev} (3.2^2) + k_{2Rev} (0.8) + k_{3Rev} (3.2) \quad \text{Eq.24}$$

At 4.5 seconds, inputting  $U=3.6 \text{ m/sec}$  and  $\frac{\partial(U)}{\partial t} = 0.8 \text{ m/sec}^2$  in Eqn (18)

$$260.956 = k_{1Rev} (3.6^2) + k_{2Rev} (0.8) + k_{2Rev} (3.6) \quad \text{Eq.25}$$

Thus, the following formula is followed for Pressure Difference between inlet and outlet for Reverse Flow Case:

$$(P_{outlet} - P_{inlet})_{Rev} = 22((U^2) + (8.5175 * (\frac{\Delta U}{\Delta t})) - 8.57125(U)) \quad \text{Eq.26}$$

Verifying the Eqn (26) at  $t=3$  seconds, where  $U$  is at  $2.4 \text{ m/sec}$ , and  $\frac{\Delta U}{\Delta t} = 0.8 \text{ m/sec}^2$

$(P_{outlet} - P_{inlet})_{Rev} = 112.9 \text{ m}^2/\text{sec}^2$  which is close to the observed value of  $114.866 \text{ m}^2/\text{sec}^2$ .

Verifying the Eqn (26) at  $t = 8 \text{ seconds}$ , where  $U$  is at  $4.8 \text{ m/sec}$ , and  $\frac{\Delta U}{\Delta t} = 1.6 \text{ m/sec}^2$

$(P_{outlet} - P_{inlet})_{Rev} = 479.36 \text{ m}^2/\text{sec}^2$  which is close to the observed value of  $462.358 \text{ m}^2/\text{sec}^2$ .

Verifying the Eqn (26) at  $t = 8.5 \text{ seconds}$ , where  $U$  is at  $5.6 \text{ m/sec}$ , and  $\frac{\Delta U}{\Delta t} = 1.6 \text{ m/sec}^2$

$(P_{outlet} - P_{inlet})_{Rev} = 655.55 \text{ m}^2/\text{sec}^2$  which is close to the observed value of  $632.569 \text{ m}^2/\text{sec}^2$ .

Thus the constants  $k_1, k_2, k_3$  for forward and reverse flow have been calculated and verified as follows:

$$k_{1_{Rev}} = 22, k_{2_{Rev}} = 8.5175, k_{3_{Rev}} = -8.57125,$$

$$k_{1_{For}} = 4.928, k_{2_{For}} = 7.824, k_{3_{For}} = -1.2655$$

For these values of constants, with ramping values of 0.8 and 1.6, we observe that  $k_{1_{Rev}}$  and  $k_{1_{For}}$  dominate, which mainly contributes to the diodicity  $\left(\frac{k_{1(Rev)}}{k_{1(For)}}\right) = 4.46$ , close to the diodicity values achieved). For a ramping value that is high enough,  $k_{2_{Rev}}$  and  $k_{2_{For}}$  would dominate. In such a scenario, since the value of  $k_{2_{Rev}}$  is close to the value of  $k_{2_{For}}$  ( $k_{2_{Rev}}$  is 9% larger than  $k_{2_{For}}$ ), the Diodicity achieved would be very small. This makes sense, since at a very high ramping value, the fluid would develop a similar Pressure difference between inlet and outlet for both forward and reverse flow case.

## 6. Conclusion

In conclusion, Equation 2 stated in this Case Study, written below, was useful in understanding Steady State as well as Transient Flow through Tesla Valve.

$$(P_{outlet} - P_{inlet}) = k_1(U^2) + k_2(\partial U/\partial t) + k_3(U) \quad \text{Eq.2}$$

The term  $k_1$  is multiplied with  $U^2$ , which is representative of velocity advection term. The term  $k_2$  is multiplied with  $\left(\frac{\partial U}{\partial t}\right)$ , which is representative of variation of velocity with time. The term  $k_3$  is multiplied with  $U$ , which is representative of viscous forces. This equation can provide a good framework for understanding both Transient and Steady state performance for incompressible, turbulent flow of Newtonian fluids through Tesla Valve. It can help understand how the diodicity will vary based on different scenarios, and the significance and effect of each term on the diodicity as the inlet flow conditions change.

The observations made in this project with respect to unsteady flow process were illuminating.

In Case 2 of the Transient Flow Case Study (Pulsating Inlet flow), it was observed that for two tesla valves at forward and reverse state connected at a common junction, any small, high frequency fluctuations in velocity, about its mean value at the common junction will be dampened and will have a small effect on the developed pressure differential. This means that if, at a common junction tesla valve is provided with a pulsating flow, the pressure difference variation between the other two ends will be dampened. This can have application in cases like axial piston pump (bent axis design), where the small, rapid variation in outlet pressure and flow of pumped fluid may need to be reduced, and pulsating flow dampener is required. It can also have applications in pipelines where vibrations due to pressure fluctuations are undesirable as excessive vibrations or vibrations at or near resonance can cause damages to the pipe. [5]

Providing reverse and forward flow Tesla Valve at the common junction will provide necessary pulse dampening so that the flow is more smooth and ensure extended life of all the parts in the hydraulic system being served.

In Case 3 of the Transient Flow Case Study (Ramped Flow), it was observed that the variation in pressure was being transmitted ahead, in proportion with the ramping of flow rate. Therefore for high, long term change in flow rate, change in pressure and velocity at the inlet does not get dampened, and gets transmitted ahead, and only rapid changes in flow rate get dampened, and are not transmitted ahead.

Future research emphasis on this topic could focus on applications of Tesla Valve in different pulsating inlet flow cases seen in various hydraulic systems to gauge the effectiveness of different arrangements of Tesla Valves in dampening pulsating inlet flow.

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