FOSSEE CFD Report

On

Atmosphere in a differentially heated rotating annulus: Baroclinic waves

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1. Introduction

Flow in atmosphere generally occurs because of the presence of temperature gradient or density gradient. The large-scale flow in atmosphere i.e. general atmospheric circulation occurs because of temperature gradient between polar region and equator. The equator is warmer because it receives more solar insolation than the polar region. Since both the equator and polar region are in thermal equilibrium so there must be transfer of heat between these two regions. This transfer of excess heat leads to formation of a large convection cell. However, because of the earth’s rotation this single large convection cell is divided into three smaller convection cell i.e. Hadley cell, Ferrel cell and polar cell. In addition, the sense of circulation of these cells varies with the hemisphere i.e. northern hemisphere and southern hemisphere.

Because of the anti-clockwise rotation of the earth, Coriolis force acts and produces the global wind pattern. Coriolis force deflect the moving object in clockwise direction in Northern hemisphere and in a counter clockwise direction in Southern Hemisphere with respect to the direction of travel. The effect of heating and rotation play an important role in maintaining the seasonal weather in the mid-latitude.

This large-scale circulation contains a baroclinic wave, which help in transfer of heat and momentum in the atmosphere. So, to understand this phenomenon more clearly rotating differentially heated annulus apparatus is used massively across literature. This apparatus consist of a cylindrical annulus on a rotating table with inner wall cooled and outer wall heated. This setup is generally known as Hide-Mason setup. A better understanding of the behavior of rotating convection will help in knowing geophysical phenomenon.

2. Methodology

To simulate this problem we have used OpenFOAM (Open Field Manipulation and Operations), which is an open source CFD toolkit. In OpenFOAM, we modified an existing CFD solver based on Boussinesq approximation for rotating convection. In our code Navier-Stokes equations with Boussinesq approximation, are solved in non-inertial frame of reference. The additional term, which comes because of non-inertial frame of reference, are Coriolis acceleration $(2 \times \omega \times r)$ and Centrifugal acceleration $(\omega \times \omega \times r)$.

Solver modification:

The code shown below is the part of momentum equation file of the solver and shows the two additional acceleration term i.e. Coriolis force and centrifugal force, which were added for making the solver ready for non-inertial frame of reference.

```
fvVectorMatrix UEqn
(
  fvm::ddt(U)
+ fvm::div(phi, U)
+ turbulence->divDevReff(U)
+ MRF.DDt(U)
+ (2*Omega ^ U) // Coriolis force
+ (Omega ^ (Omega ^ (mesh.C()))) // Centrifugal force
==
  fvOptions(U)

 );
```

For meshing purpose, structured mesh is used and to capture the boundary layer we maintained near wall criteria. For meshing purpose, Ansys ICEM CFD is used.

3. Governing equations

The equations of motion governing the incompressible fluid of density ρ and kinematic viscosity ν in non-inertial form of reference is

$$\left[\frac{Du}{Dt} - \mathrm{f} \times v\right] = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + \nu\nabla^2 u \tag{3.1}$$

$$\left[\frac{Dv}{Dt} + \mathrm{f} \times u\right] = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{y}} + \nu\nabla^2 v \tag{3.2}$$

$$\left[\frac{Dw}{Dt}\right] = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{z}} - \mathrm{g}\frac{(\rho - \rho_0)}{\rho_0} + \nu\nabla^2 w \tag{3.3}$$

Here $u, v, \& w$ are the velocity of the fluid at a point fixed in the rotating frame, p is the pressure and g is the gravitational force. The momentum equation have Boussinesq approximation in it. According to Boussinesq approximation, the density changes in the fluid can be ignored except where density is multiplied with gravitational acceleration. This approximation also consider other fluid properties as constant. Boussinesq approximation is valid for vertical scale length of the flow less than 10 km.

The continuity of matter requires that

$$\nabla . \boldsymbol{u} = 0 \tag{3.4}$$

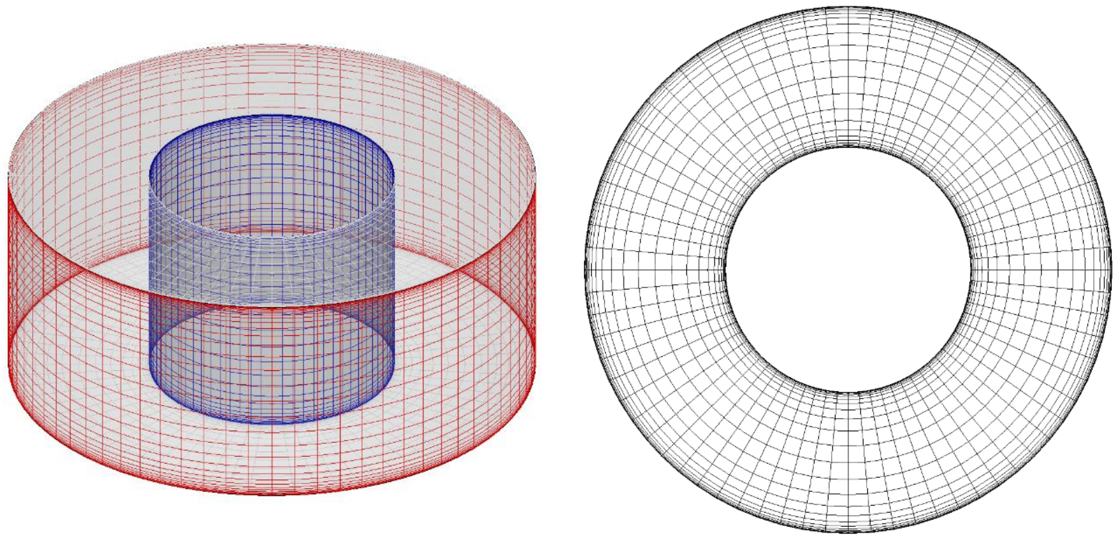
Energy equation for heat transfer is

$$\frac{DT}{Dt} = \kappa \nabla^2 T \tag{3.5}$$

Here T is the temperature of the fluid.

4. Test case description

Figure 1 shows the geometry of test case (taken from Ukaji et. al. 1989). The outer radius, inner radius and height of cylinder are 9.712 cm, 4.512 cm and 8 cm respectively. The temperature of inner and outer wall are 25℃ and 28℃ respectively. The fluid used is water $\mathrm{Pr} = 5.98$. Figure 1 shows the meshing of the geometry with 25, 80 and 35 in $r, \theta\ and\ z$ direction respectively. At outer wall and inner wall constant temperature boundary condition is given and at rest, walls adiabatic boundary condition is given. For velocity boundary condition, no-slip condition is used. For stability of solution courant number is used and maximum courant number is set to 0.5. All simulation are done at $Ra = 5.89 \times 10^6$ and varying Taylor number i.e. $Ta = 6.35 \times 10^6, 9.14 \times 10^6, 1.07 \times 10^7\ and\ 1.24 \times 10^7$.



(a) (b)

Figure 1: (a) 3D view of annular geometry (Ukaji et. al 1989) with meshing, Red colors shows heating wall and blue color shows cold wall. (b) Top view of the geometry with mesh.

5. Result and discussion

5.1 Temperature contours

Figure 2 shows the temperature contours on $r-\theta$ plane at $z=4cm$ from the bottom, for $Ra = 5.89 \times 10^6$ and $Ta = 6.35 \times 10^6, 9.14 \times 10^6, 1.07 \times 10^7$ and 1.24×10^7. The temperature contours shows the presence of the baroclinic wave in the bulk of the fluid. This wave is present throughout the height of the annulus. We can observe that the baroclinic wave is forming from the axisymmetric flow regime and getting into wave flow regime, as we move from lower Taylor number to higher Taylor number. The baroclinicity happens because of the sloping of the temperature isotherms, which leads to release of the potential energy into kinetic energy and so the baroclinic wave forms. The inclined isotherms are shown in Figure 3.

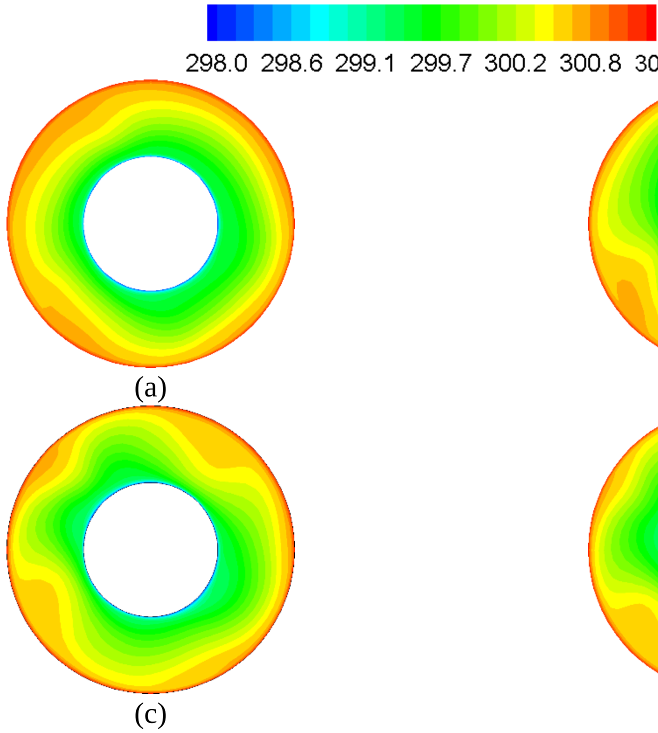


Figure 2: Temeprature contours at $z = 4cm$ from bottom for $Ra = 5.89 \times 10^6$ and varrying Ta (a) 6.35×10^6, (b) 9.14×10^6, (c) 1.07×10^7 and (d) 1.24×10^7.

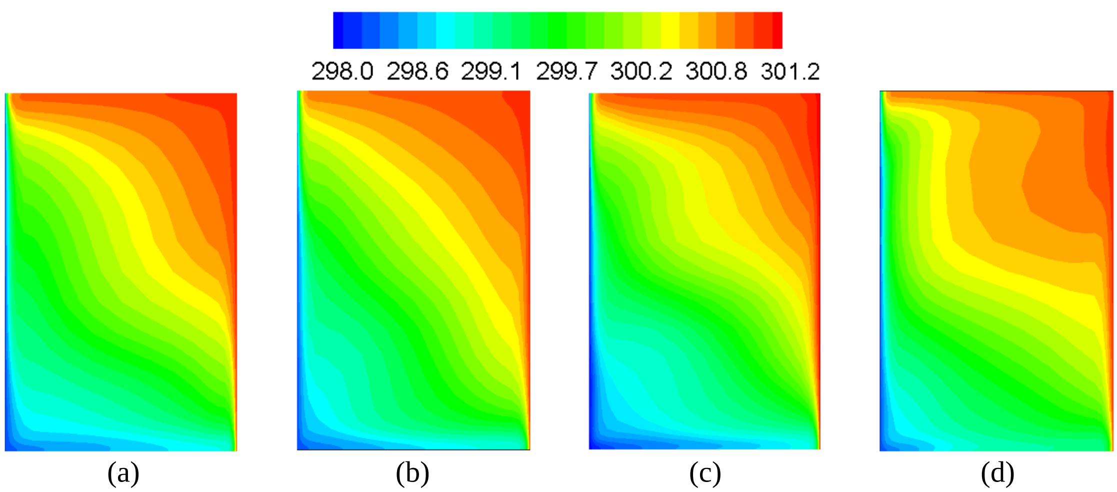


Figure 3: Temperature contours on $r - z$ plane for $Ra = 5.89 \times 10^6$ and varrying Ta (a) 6.35×10^6, (b) 9.14×10^6, (c) 1.07×10^7 and (d) 1.24×10^7.

5.2 Horizontal velocity contours

Horizontal velocity is the magnitude of u_r and u_θ components of velocity. Figure 4 shows the contours of horizontal velocity at $z = 7.5cm$ from the bottom. From the contours, it is observed that there is dominant flow in the middle region of the annular gap. The flow near the boundary wall is affected by the boundary layers. From figure 5, it was observed that as the Taylor number increase the flow is being more aligned with baroclinic wave.

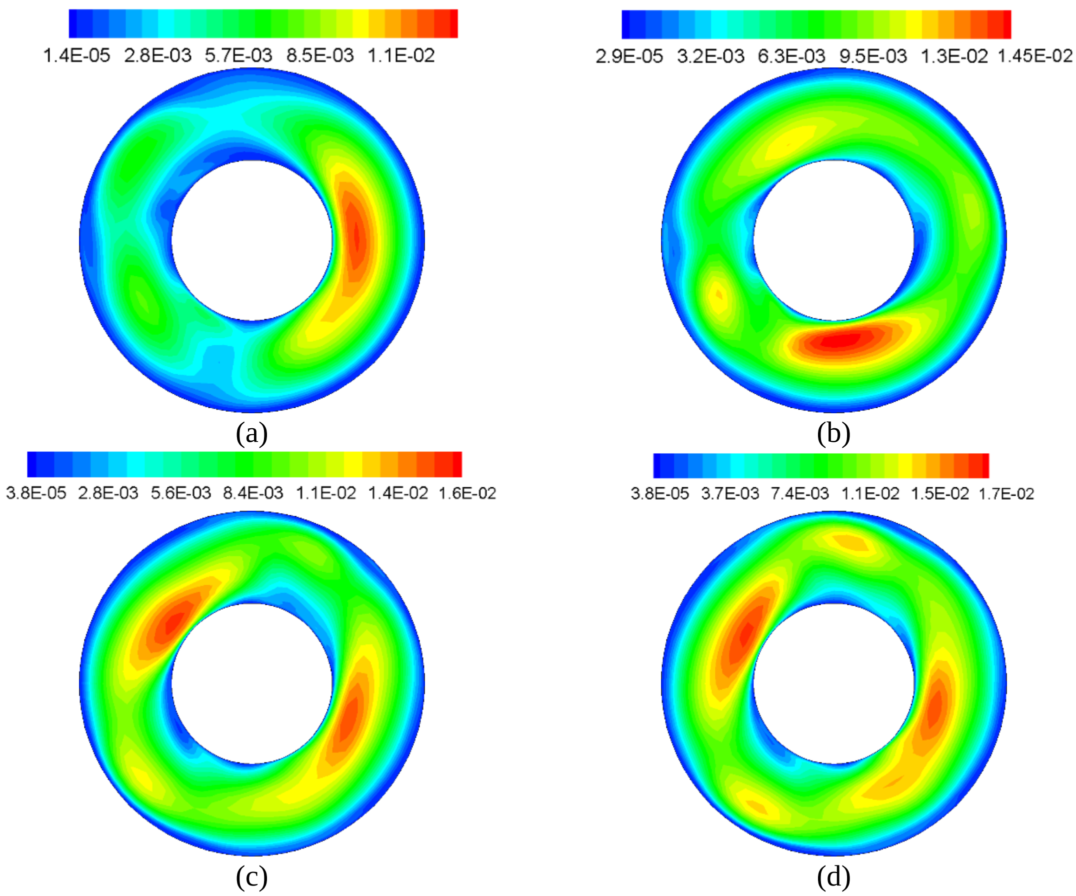


Figure 4: Horizontal velocity contours on $r - z$ plane at $z = 7.5cm$ for $Ra = 5.89 \times 10^6$ and varrying Ta (a) 6.35×10^6, (b) 9.14×10^6, (c) 1.07×10^7 and (d) 1.24×10^7.

6. Conclusions

Geophysical phenomenon i.e. general atmospheric circulation occurs because of the temperature difference between equator and polar region. This phenomenon can be studied in experiments or simulation in differentially heated rotating annulus apparatus. From our simulation of general atmospheric circulation at $Ra = 5.89 \times 10^6$and various, $Ta = 6.35 \times 10^6, 9.14 \times 10^6, 1.07 \times 10^7$ and 1.24×10^7 and founded that the flow is in transition from axisymmetric flow to wave flow regime. It was also observed that the slope of the isotherms also change as the Taylor number increases.

7. Reference

Ukaji, Kazuo, and Katsumi Tamaki. "A comparison of laboratory experiments and numerical simulations of steady baroclinic waves produced in a differentially heated rotating fluid annulus." *Journal of the Meteorological Society of Japan. Ser. II* 67, no. 3 (1989): 359-374.